

linear and multilinear algebra

Linear and Multilinear Algebra: A Comprehensive Guide

Introduction:

Are you ready to unlock the secrets of vector spaces, matrices, and tensors? This comprehensive guide dives deep into the fascinating worlds of linear and multilinear algebra, providing a clear and accessible explanation of fundamental concepts and their applications. Whether you're a student grappling with challenging coursework, a researcher needing a solid foundation, or simply a curious mind seeking to understand the mathematical underpinnings of numerous fields, this post offers a structured exploration, addressing key concepts with practical examples and insightful explanations. We'll cover the core tenets of linear algebra, transition seamlessly into the more complex realm of multilinear algebra, and highlight the crucial connections between them. Get ready to expand your mathematical horizons!

1. Linear Algebra: The Foundation

Linear algebra forms the bedrock upon which much of modern mathematics and its applications are built. At its heart lies the concept of a vector space, a collection of objects (vectors) that can be added together and multiplied by scalars (usually real or complex numbers) while obeying specific rules. These rules ensure the closure of the vector space under addition and scalar multiplication. Key aspects of linear algebra include:

Vectors and Vector Spaces: Understanding the properties of vectors, their representation (e.g., as column matrices), and the different types of vector spaces (e.g., Euclidean spaces, function spaces). We'll explore concepts like linear independence, spanning sets, and bases, which are crucial for understanding the structure of vector spaces.

Matrices and Linear Transformations: Matrices provide a powerful tool for representing linear transformations – functions that map vectors from one vector space to another while preserving linear combinations. We'll cover matrix operations (addition, multiplication, transpose, inverse), matrix rank, and determinants, and explore their significance in solving systems of linear equations.

Eigenvalues and Eigenvectors: These are fundamental concepts in linear algebra, representing special vectors that remain unchanged (up to scaling) when a linear transformation is applied. Understanding eigenvalues and eigenvectors is crucial in numerous applications, including stability analysis in dynamical systems and dimensionality reduction techniques.

Systems of Linear Equations: Solving systems of linear equations is a cornerstone application of linear algebra. We will cover various methods, such as Gaussian elimination, LU decomposition, and the use of matrix inverses. Understanding the different cases (unique solution, infinitely many

solutions, no solution) is critical.

1. Stepping into Multilinear Algebra: Beyond Linearity

Multilinear algebra extends the concepts of linear algebra by considering functions that act on multiple vectors simultaneously. Instead of linear transformations mapping single vectors, we deal with multilinear maps, also known as tensors. These maps exhibit linearity in each of their vector arguments independently. Key concepts in multilinear algebra include:

Tensors: A Generalization of Vectors and Matrices: Tensors can be viewed as a generalization of vectors and matrices. A vector is a first-order tensor, a matrix is a second-order tensor, and higher-order tensors represent multilinear relationships involving more than two vectors.

Tensor Products: The tensor product is a fundamental operation in multilinear algebra that combines multiple vectors or tensors to create a higher-order tensor. Understanding the tensor product is vital for constructing and manipulating tensors effectively.

Tensor Contraction: This operation involves summing over pairs of indices in a tensor, effectively reducing its order. It plays a significant role in various applications, such as calculating inner products and performing matrix multiplications.

Metric Tensors and Inner Products: The metric tensor is a crucial element in defining inner products (dot products) in more general vector spaces than Euclidean space. It allows us to generalize notions of length, distance, and angles to different geometric settings.

Applications of Multilinear Algebra: Multilinear algebra finds numerous applications in diverse fields, including machine learning (tensor decompositions, tensor networks), physics (general relativity, quantum mechanics), and computer graphics (texture mapping, image processing).

1. The Interplay Between Linear and Multilinear Algebra:

Linear and multilinear algebra are intrinsically linked. Linear algebra provides the foundational concepts and tools that are essential for understanding and working with multilinear objects. The properties of vectors and matrices, the concepts of linear independence and bases, and the techniques for solving linear equations all play critical roles in the development and application of multilinear algebra. Moreover, many concepts in multilinear algebra can be viewed as generalizations of their linear algebra counterparts, enriching our understanding of both areas.

1. A Textbook Example Outline: "Understanding Linear and Multilinear Algebra"

Title: Understanding Linear and Multilinear Algebra

Outline:

Introduction: Overview of linear and multilinear algebra, its importance, and the scope of the book.

Chapter 1: Fundamentals of Linear Algebra: Vectors, vector spaces, linear independence, spanning sets, bases, linear transformations, matrices, matrix operations.

Chapter 2: Linear Equations and their Solutions: Systems of linear equations, Gaussian elimination, LU decomposition, matrix inverses, eigenvalues and eigenvectors.

Chapter 3: Introduction to Multilinear Algebra: Tensors, tensor products, tensor contraction, metric tensors, inner products.

Chapter 4: Applications of Linear and Multilinear Algebra: Examples in physics, computer science, machine learning, and engineering.

Conclusion: Summary of key concepts, further reading suggestions.

Detailed Explanation of Outline Points:

(Elaboration on each point would require expanding this to book length. The following are brief expansions to exemplify the level of detail.)

Introduction: This chapter would provide a motivating overview, highlighting the importance of linear and multilinear algebra in various fields and outlining the book's structure and approach. It would aim to make the subject accessible to a broad audience.

Chapter 1: This chapter would build a solid foundation in linear algebra, starting with the definition of vectors and vector spaces, moving onto linear combinations, independence, and bases. It would then introduce matrices as representations of linear transformations and explore various matrix operations and their properties.

Chapter 2: This chapter would delve into the practical application of linear algebra: solving systems of linear equations. It would cover various methods, like Gaussian elimination and LU decomposition, and explore the concepts of eigenvalues and eigenvectors, explaining their importance in various contexts.

Chapter 3: This chapter would transition into the realm of multilinear algebra, introducing tensors as generalizations of vectors and matrices. It would explain tensor products and contractions, showing how these operations manipulate tensors. The chapter would also explore metric tensors and their role in defining inner products.

Chapter 4: This chapter would demonstrate the wide-ranging applicability of both linear and multilinear algebra through concrete examples from diverse fields. It would use illustrative examples to show how the concepts discussed earlier are used in practical scenarios.

Conclusion: The concluding chapter would synthesize the key ideas of the book, reiterating the fundamental concepts and highlighting the connections between linear and multilinear algebra. It would also point readers towards further learning resources and advanced topics.

FAQs:

1. What is the difference between linear and multilinear algebra? Linear algebra deals with single vectors and linear transformations, while multilinear algebra studies multilinear maps and tensors, which act on multiple vectors simultaneously.
1. Why is linear algebra important? It's foundational for many fields, providing tools to solve systems of equations, analyze data, and model various phenomena.
1. What are tensors used for? Tensors are crucial in machine learning, physics (general relativity), computer graphics, and more, representing multidimensional data and relationships.
1. How are eigenvalues and eigenvectors used? They are used in analyzing matrices, stability of systems, and dimensionality reduction techniques.
1. What is a tensor product? It's an operation that combines tensors to create a higher-order tensor, forming a fundamental building block of multilinear algebra.
1. What is tensor contraction? It's a method to reduce the order of a tensor by summing over indices, simplifying computations and extracting information.
1. Is multilinear algebra harder than linear algebra? Yes, multilinear algebra builds upon linear algebra and introduces more abstract concepts, making it generally more challenging.
1. Where can I learn more about linear algebra? Numerous textbooks and online courses are available, catering to various levels of mathematical background.
1. What are some real-world applications of multilinear algebra? Image processing, recommendation systems, quantum computing, and fluid dynamics utilize multilinear algebra concepts.

Related Articles:

1. Introduction to Vector Spaces: A beginner-friendly explanation of vector spaces, their properties, and examples.
1. Matrix Operations and their Applications: A deep dive into matrix algebra and its use in solving linear equations.
1. Eigenvalues and Eigenvectors: A Practical Guide: A comprehensive tutorial on eigenvalues and eigenvectors, their calculation, and interpretations.
1. Understanding Linear Transformations: An exploration of linear transformations, their properties, and their representation using matrices.
1. Introduction to Tensors: A Gentle Approach: An accessible introduction to tensors, their representation, and basic operations.
1. Tensor Products and their Significance: A detailed explanation of the tensor product operation and its applications.
1. Tensor Contraction: Simplifying Tensor Calculations: A guide to tensor contraction and its role in reducing tensor complexity.
1. Applications of Linear Algebra in Machine Learning: An overview of how linear algebra underpins various machine learning algorithms.
1. Multilinear Algebra in Physics: A Case Study: An exploration of the use of multilinear algebra

in solving problems in physics, such as general relativity.

linear and multilinear algebra: Multilinear Algebra Werner H. Greub, 2012-12-06 This book is built around the material on multilinear algebra which in chapters VI to IX of the second edition of Linear Algebra was included but excluded from the third edition. It is designed to be a sequel and companion volume to the third edition of Linear Algebra. In fact, the terminology and basic results of that book are frequently used without reference. In particular, the reader should be familiar with chapters I to V and the first part of chapter VI although other sections are occasionally used. The essential difference between the present treatment and that of the second edition lies in the full exploitation of universal properties which eliminates the restriction to vector spaces of finite dimension. Chapter I contains standard material on multilinear mappings and the tensor product of vector spaces. These results are extended in Chapter 11 to vector spaces with additional structure, such as algebras and differential spaces. The fundamental concept of tensor product is used in Chapter 11 to construct the tensor algebra over a given vector space. In the next chapter the link is provided between tensor algebra on the one hand and exterior and symmetric tensor algebra on the other. Chapter V contains material on exterior algebra which is developed in considerable depth. Exterior algebra techniques are used in the following chapter as a powerful tool to obtain matrix-free proofs of many classical theorems on linear transformation.

linear and multilinear algebra: Multilinear Algebra Werner Greub, 2012-12-06 This book is a revised version of the first edition and is intended as a Linear Algebra sequel and companion volume to the fourth edition of (Graduate Texts in Mathematics 23). As before, the terminology and basic results of Linear Algebra are frequently used without reference. In particular, the reader should be familiar with Chapters 1-5 and the first part of Chapter 6 of that book, although other sections are occasionally used. In this new version of Multilinear Algebra, Chapters 1-5 remain essentially unchanged from the previous edition. Chapter 6 has been completely rewritten and split into three (Chapters 6, 7, and 8). Some of the proofs have been simplified and a substantial amount of new material has been added. This applies particularly to the study of characteristic coefficients and the Pfaffian. The old Chapter 7 remains as it stood, except that it is now Chapter 9. The old Chapter 8 has been suppressed and the material which it contained (multilinear functions) has been relocated at the end of Chapters 3, 5, and 9. The last two chapters on Clifford algebras and their representations are completely new. In view of the growing importance of Clifford algebras and the relatively few references available, it was felt that these chapters would be useful to both mathematicians and physicists.

linear and multilinear algebra: Elements Of Linear And Multilinear Algebra John M Erdman, 2020-12-22 This set of notes is an activity-oriented introduction to linear and multilinear algebra. The great majority of the most elementary results in these subjects are straightforward and can be verified by the thoughtful student. Indeed, that is the main point of these notes — to convince the beginner that the subject is accessible. In the material that follows there are numerous indicators that suggest activity on the part of the reader: words such as 'proposition', 'example', 'theorem', 'exercise', and 'corollary', if not followed by a proof (and proofs here are very rare) or a reference to a proof, are invitations to verify the assertions made. These notes are intended to accompany an (academic) year-long course at the advanced undergraduate or beginning graduate level. (With judicious pruning most of the material can be covered in a two-term sequence.) The text is also suitable for a lecture-style class, the instructor proving some of the results while leaving others as exercises for the students. This book has tried to keep the facts about vector spaces and those about inner product spaces separate. Many beginning linear algebra texts conflate the material on these two vastly different subjects.

linear and multilinear algebra: Multilinear Algebra Russell Merris, 1997-08-20 The prototypical multilinear operation is multiplication. Indeed, every multilinear mapping can be factored through a tensor product. Apart from its intrinsic interest, the tensor product is of fundamental importance in a variety of disciplines, ranging from matrix inequalities and group representation theory, to the combinatorics of symmetric func

linear and multilinear algebra: Tensors: Geometry and Applications J. M. Landsberg, 2011-12-14 Tensors are ubiquitous in the sciences. The geometry of tensors is both a powerful tool for extracting information from data sets, and a beautiful subject in its own right. This book has three intended uses: a classroom textbook, a reference work for researchers in the sciences, and an account of classical and modern results in (aspects of) the theory that will be of interest to researchers in geometry. For classroom use, there is a modern introduction to multilinear algebra and to the geometry and representation theory needed to study tensors, including a large number of exercises. For researchers in the sciences, there is information on tensors in table format for easy reference and a summary of the state of the art in elementary language. This is the first book containing many classical results regarding tensors. Particular applications treated in the book include the complexity of matrix multiplication, P versus NP, signal processing, phylogenetics, and algebraic statistics. For geometers, there is material on secant varieties, G-varieties, spaces with finitely many orbits and how these objects arise in applications, discussions of numerous open questions in geometry arising in applications, and expositions of advanced topics such as the proof of the Alexander-Hirschowitz theorem and of the Weyman-Kempf method for computing syzygies.

linear and multilinear algebra: Introduction to Vectors and Tensors Ray M. Bowen, Chao-cheng Wang, 1976-05-31 To Volume 1 This work represents our effort to present the basic concepts of vector and tensor analysis. Volume 1 begins with a brief discussion of algebraic structures followed by a rather detailed discussion of the algebra of vectors and tensors. Volume 2 begins with a discussion of Euclidean manifolds, which leads to a development of the analytical and geometrical aspects of vector and tensor fields. We have not included a discussion of general differentiable manifolds. However, we have included a chapter on vector and tensor fields defined on hypersurfaces in a Euclidean manifold. In preparing this two-volume work, our intention was to present to engineering and science students a modern introduction to vectors and tensors. Traditional courses on applied mathematics have emphasized problem-solving techniques rather than the systematic development of concepts. As a result, it is possible for such courses to become terminal mathematics courses rather than courses which equip the student to develop his or her understanding further.

linear and multilinear algebra: Introduction to Linear and Matrix Algebra Nathaniel Johnston, 2021-05-19 This textbook emphasizes the interplay between algebra and geometry to motivate the study of linear algebra. Matrices and linear transformations are presented as two sides of the same coin, with their connection motivating inquiry throughout the book. By focusing on this interface, the author offers a conceptual appreciation of the mathematics that is at the heart of further theory and applications. Those continuing to a second course in linear algebra will appreciate the companion volume *Advanced Linear and Matrix Algebra*. Starting with an introduction to vectors, matrices, and linear transformations, the book focuses on building a geometric intuition of what these tools represent. Linear systems offer a powerful application of the ideas seen so far, and lead onto the introduction of subspaces, linear independence, bases, and rank. Investigation then focuses on the algebraic properties of matrices that illuminate the geometry of the linear transformations that they represent. Determinants, eigenvalues, and eigenvectors all benefit from this geometric viewpoint. Throughout, "Extra Topic" sections augment the core content with a wide range of ideas and applications, from linear programming, to power iteration and linear recurrence relations. Exercises of all levels accompany each section, including many designed to be tackled using computer software. *Introduction to Linear and Matrix Algebra* is ideal for an introductory proof-based linear algebra course. The engaging color presentation and frequent marginal notes showcase the author's visual approach. Students are assumed to have completed one

or two university-level mathematics courses, though calculus is not an explicit requirement. Instructors will appreciate the ample opportunities to choose topics that align with the needs of each classroom, and the online homework sets that are available through WeBWork.

linear and multilinear algebra: Algebra: Chapter 0 Paolo Aluffi, 2021-11-09 Algebra: Chapter 0 is a self-contained introduction to the main topics of algebra, suitable for a first sequence on the subject at the beginning graduate or upper undergraduate level. The primary distinguishing feature of the book, compared to standard textbooks in algebra, is the early introduction of categories, used as a unifying theme in the presentation of the main topics. A second feature consists of an emphasis on homological algebra: basic notions on complexes are presented as soon as modules have been introduced, and an extensive last chapter on homological algebra can form the basis for a follow-up introductory course on the subject. Approximately 1,000 exercises both provide adequate practice to consolidate the understanding of the main body of the text and offer the opportunity to explore many other topics, including applications to number theory and algebraic geometry. This will allow instructors to adapt the textbook to their specific choice of topics and provide the independent reader with a richer exposure to algebra. Many exercises include substantial hints, and navigation of the topics is facilitated by an extensive index and by hundreds of cross-references.

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linear and multilinear algebra: Linear Algebra and Geometry Igor R. Shafarevich, Alexey O. Remizov, 2012-08-23 This book on linear algebra and geometry is based on a course given by renowned academician I.R. Shafarevich at Moscow State University. The book begins with the theory of linear algebraic equations and the basic elements of matrix theory and continues with vector spaces, linear transformations, inner product spaces, and the theory of affine and projective spaces. The book also includes some subjects that are naturally related to linear algebra but are usually not covered in such courses: exterior algebras, non-Euclidean geometry, topological properties of projective spaces, theory of quadrics (in affine and projective spaces), decomposition of finite abelian groups, and finitely generated periodic modules (similar to Jordan normal forms of linear operators). Mathematical reasoning, theorems, and concepts are illustrated with numerous examples from various fields of mathematics, including differential equations and differential geometry, as well as from mechanics and physics.

linear and multilinear algebra: Linear Algebra Done Right Sheldon Axler, 1997-07-18 This text for a second course in linear algebra, aimed at math majors and graduates, adopts a novel approach by banishing determinants to the end of the book and focusing on understanding the structure of linear operators on vector spaces. The author has taken unusual care to motivate concepts and to simplify proofs. For example, the book presents - without having defined determinants - a clean proof that every linear operator on a finite-dimensional complex vector space has an eigenvalue. The book starts by discussing vector spaces, linear independence, span, basics, and dimension. Students are introduced to inner-product spaces in the first half of the book and shortly thereafter to the finite-dimensional spectral theorem. A variety of interesting exercises in each chapter helps students understand and manipulate the objects of linear algebra. This second edition features new chapters on diagonal matrices, on linear functionals and adjoints, and on the spectral theorem; some sections, such as those on self-adjoint and normal operators, have been entirely rewritten; and hundreds of minor improvements have been made throughout the text.

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linear and multilinear algebra: Multilinear Algebra D. G. Northcott, 2009-01-18 Multilinear algebra has important applications in many different areas of mathematics but is usually learned in a rather haphazard fashion. The aim of this book is to provide a readable and systematic account of

multilinear algebra at a level suitable for graduate students. Professor Northcott gives a thorough treatment of topics such as tensor, exterior, Grassmann, Hopf and co-algebras and ends each chapter with a section entitled 'Comments and Exercises'. The comments contain convenient summaries and discussion of the content whilst the exercises provide an opportunity to test understanding and add extra material. Complete solutions are provided for those exercises that are particularly important or used later in the book. The volume as a whole is based on advanced lectures given by the author at the University of Sheffield.

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linear and multilinear algebra: Matrix Theory Robert Piziak, P.L. Odell, 2007-02-22 In 1990, the National Science Foundation recommended that every college mathematics curriculum should include a second course in linear algebra. In answer to this recommendation, *Matrix Theory: From Generalized Inverses to Jordan Form* provides the material for a second semester of linear algebra that probes introductory linear algebra concepts while also exploring topics not typically covered in a sophomore-level class. Tailoring the material to advanced undergraduate and beginning graduate students, the authors offer instructors flexibility in choosing topics from the book. The text first focuses on the central problem of linear algebra: solving systems of linear equations. It then discusses LU factorization, derives Sylvester's rank formula, introduces full-rank factorization, and describes generalized inverses. After discussions on norms, QR factorization, and orthogonality, the authors prove the important spectral theorem. They also highlight the primary decomposition theorem, Schur's triangularization theorem, singular value decomposition, and the Jordan canonical

form theorem. The book concludes with a chapter on multilinear algebra. With this classroom-tested text students can delve into elementary linear algebra ideas at a deeper level and prepare for further study in matrix theory and abstract algebra.

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linear and multilinear algebra: *Linear and Multilinear Algebra and Function Spaces* A. Bourhim, J. Mashreghi, L. Oubbi, Z. Abdelali, 2020-05-05 This volume contains the proceedings of the International Conference on Algebra and Related Topics, held from July 2-5, 2018, at Mohammed V University, Rabat, Morocco. Linear reserver problems demand the characterization of linear maps between algebras that leave invariant certain properties or certain subsets or relations. One of the most intractable unsolved problems is Kaplansky's conjecture: every surjective unital invertibility preserving linear map between two semisimple Banach algebras is a Jordan homomorphism. Recently, there has been an upsurge of interest in nonlinear preservers, where the maps studied are no longer assumed linear but instead a weak algebraic condition is somehow involved through the preserving property. This volume contains several articles on various aspects of preservers, including such topics as Jordan isomorphisms, Aluthge transform, joint numerical radius on $C^* \times C^*$ -algebras, advertible complete algebras, and Gelfand-Mazur algebras. The volume also contains a survey on recent progress on local spectrum-preserving maps. Several articles in the volume present results about weighted spaces and algebras of holomorphic or harmonic functions, including biduality in weighted spaces of analytic functions, interpolation in the analytic Wiener algebra, and weighted composition operators on non-locally convex weighted spaces.

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linear and multilinear algebra: *Advanced Calculus (Revised Edition)* Lynn Harold Loomis, Shlomo Zvi Sternberg, 2014-02-26 An authorised reissue of the long out of print classic textbook, *Advanced Calculus* by the late Dr Lynn Loomis and Dr Shlomo Sternberg both of Harvard University has been a revered but hard to find textbook for the advanced calculus course for decades. This book is based on an honors course in advanced calculus that the authors gave in the 1960's. The foundational material, presented in the unstarred sections of Chapters 1 through 11, was normally covered, but different applications of this basic material were stressed from year to year, and the book therefore contains more material than was covered in any one year. It can accordingly be used (with omissions) as a text for a year's course in advanced calculus, or as a text for a three-semester introduction to analysis. The prerequisites are a good grounding in the calculus of one variable from a mathematically rigorous point of view, together with some acquaintance with linear algebra. The reader should be familiar with limit and continuity type arguments and have a certain amount of mathematical sophistication. As possible introductory texts, we mention *Differential and Integral Calculus* by R Courant, *Calculus* by T Apostol, *Calculus* by M Spivak, and *Pure Mathematics* by G Hardy. The reader should also have some experience with partial derivatives. In overall plan the book divides roughly into a first half which develops the calculus (principally the differential calculus) in the setting of normed vector spaces, and a second half which deals with the calculus of differentiable

manifolds.

linear and multilinear algebra: Introduction To Linear Algebra Mark J. DeBonis, 2022-02-23 Introduction to Linear Algebra: Computation, Application, and Theory is designed for students who have never been exposed to the topics in a linear algebra course. The text is filled with interesting and diverse application sections but is also a theoretical text which aims to train students to do succinct computation in a knowledgeable way. After completing the course with this text, the student will not only know the best and shortest way to do linear algebraic computations but will also know why such computations are both effective and successful. Features: Includes cutting edge applications in machine learning and data analytics Suitable as a primary text for undergraduates studying linear algebra Requires very little in the way of pre-requisites

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linear and multilinear algebra: Functional Linear Algebra Hannah Robbins, 2021-04-06 Linear algebra is an extremely versatile and useful subject. It rewards those who study it with powerful computational tools, lessons about how mathematical theory is built, examples for later study in other classes, and much more. Functional Linear Algebra is a unique text written to address the need for a one-term linear algebra course where students have taken only calculus. It does not assume students have had a proofs course. The text offers the following approaches: More emphasis is placed on the idea of a linear function, which is used to motivate the study of matrices and their operations. This should seem natural to students after the central role of functions in calculus. Row reduction is moved further back in the semester and vector spaces are moved earlier to avoid an artificial feeling of separation between the computational and theoretical aspects of the course. Chapter 0 offers applications from engineering and the sciences to motivate students by revealing how linear algebra is used. Vector spaces are developed over $\mathbb{R}$, but complex vector spaces are discussed in Appendix A.1. Computational techniques are discussed both by hand and using technology. A brief introduction to Mathematica is provided in Appendix A.2. As readers work through this book, it is important to understand the basic ideas, definitions, and computational skills. Plenty of examples and problems are provided to make sure readers can practice until the material is thoroughly grasped. Author Dr. Hannah Robbins is an associate professor of mathematics at Roanoke College, Salem, VA. Formerly a commutative algebraist, she now studies applications of linear algebra and assesses teaching practices in calculus. Outside the office, she enjoys hiking and playing bluegrass bass.

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used with a browser with Java capabilities) and MATLAB® coding. This allows students to discover the theory and applications of linear algebra in a meaningful and memorable way. Features: The book's inquiry-based approach promotes student interaction Each chapter contains a project set which consists of application-driven projects emphasizing the chapter's materials Adds a project component to any Linear Algebra course Explores many applications to a variety of fields that can promote research projects Employs MATLAB® to calculate and explore concepts and theories of linear algebra

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exercises start from easy computations, such as computing determinants of matrices, to practical applications on simulated and empirical data sets with R so that students learn how to get started with R, along with computational examples in each section, and then students learn how to apply what they've learned to problems in applied sciences. This book is designed from first principles to demonstrate the importance of linear algebra through working computational examples with R and Python, including tutorials on how to install R in the Appendix. If a student has never seen R, they can get started without any additional help. Since Python is one of the most popular languages in data science, optimization, and computer science, code supplements are available for students who feel more comfortable with Python. R is used primarily for computational examples to develop students' practical computational skills. About the Author: Dr. Ruriko Yoshida is an Associate Professor of Operations Research at the Naval Postgraduate School. She received her PhD in Mathematics from the University of California, Davis. Her research topics cover a wide variety of areas: applications of algebraic combinatorics to statistical problems such as statistical learning on non-Euclidean spaces, sensor networks, phylogenetics, and phylogenomics. She teaches courses in statistics, stochastic models, probability, and data science.

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