

# neukirch algebraic number theory

## Delving Deep into Neukirch's Algebraic Number Theory: A Comprehensive Guide

### Introduction:

Are you fascinated by the intricate world of numbers beyond the familiar integers and rationals? Do you yearn to understand the profound connections between algebra and number theory? Then Jürgen Neukirch's Algebraic Number Theory is your essential guide. This comprehensive post will serve as your roadmap, navigating the core concepts, key theorems, and overall structure of this seminal text. We'll explore its essential components, highlighting the depth and breadth of Neukirch's masterful work and providing you with the tools to appreciate its profound impact on the field. Get ready to embark on a journey into the elegant and challenging world of algebraic number theory!

### Chapter 1: Laying the Foundations – Basic Concepts and Prerequisites

Neukirch's book doesn't shy away from rigor. It begins by establishing a solid foundation in abstract algebra and field theory. This isn't just a cursory review; it delves into crucial concepts that are fundamental to understanding algebraic number theory. Key areas covered include:

**Groups:** The text thoroughly covers group theory, essential for understanding Galois theory, a cornerstone of algebraic number theory. This includes topics like subgroups, quotient groups, group actions, and Sylow theorems. Understanding these concepts is critical for grasping the relationships between fields and their Galois groups.

**Rings and Fields:** A solid understanding of ring theory, including ideals, prime ideals, and quotient rings, is paramount. The book rigorously lays out these concepts, providing the necessary framework for understanding algebraic number fields. The exploration of field extensions, particularly finite extensions, is central to the development of the theory.

**Modules:** The theory of modules over rings plays a vital role in understanding the structure of ideals in number fields. Neukirch's treatment of modules prepares the reader for the sophisticated arguments that follow.

### Chapter 2: Algebraic Number Fields – Delving into the Heart of the Matter

This chapter marks the true beginning of algebraic number theory. Here, Neukirch introduces the core objects of study: algebraic number fields. Key concepts explored include:

**Field Extensions:** The book explores the properties of finite extensions of the rational numbers, which are the algebraic number fields. The notions of degree, minimal polynomial, and embeddings are meticulously explained.

**Ring of Integers:** The concept of the ring of integers within an algebraic number field is central. Neukirch clarifies the structure and properties of this ring, a key element in

understanding ideal theory.

**Ideals and Factorization:** This is where the magic happens. The unique factorization of integers in the rational numbers breaks down in algebraic number fields. Neukirch introduces the concept of ideals as a replacement for prime factorization, leading to unique factorization of ideals.

### Chapter 3: Class Field Theory – The Crown Jewel of Algebraic Number Theory

Class field theory represents one of the pinnacles of algebraic number theory. It establishes a profound connection between the arithmetic of a number field and its Galois group. Neukirch's treatment of this topic is renowned for its clarity and depth. Key elements covered include:

**Ideal Class Group:** This group measures the failure of unique factorization in an algebraic number field. Understanding its structure is crucial for understanding class field theory.

**Hilbert Class Field:** This field is a fundamental object in class field theory. Neukirch explores its properties and its relationship to the ideal class group.

**Artin Reciprocity Law:** This is the central theorem of class field theory. It establishes a deep and unexpected connection between the arithmetic of a number field and its Galois group. Neukirch's explanation of this theorem is considered a model of clarity.

### Chapter 4: Local Fields and p-adic Numbers – A Different Perspective

Neukirch also explores the theory of local fields, which offer a different but equally crucial perspective on algebraic number theory. This chapter introduces:

**p-adic Numbers:** These numbers provide a different way of looking at the rational numbers, focusing on their behaviour modulo powers of a prime  $p$ .

**Local Fields:** These are complete fields with respect to a non-archimedean valuation, providing a powerful tool for studying algebraic number fields.

**Local-Global Principles:** This principle connects the local properties of a number field (at each prime) to its global properties.

### Chapter 5: Cohomology of Groups and Applications – Advanced Tools and Techniques

The final chapters delve into more advanced techniques, notably the cohomology of groups, which plays a significant role in modern algebraic number theory.

**Group Cohomology:** Neukirch introduces the basic concepts and results of group cohomology, providing the necessary tools for understanding more advanced aspects of class field theory.

**Applications to Class Field Theory:** The cohomology of groups is used to provide elegant proofs and generalizations of important results in class field theory.

### Book Outline: Neukirch's Algebraic Number Theory

Title: Algebraic Number Theory

Author: Jürgen Neukirch

## Contents:

Introduction: Overview of algebraic number theory and its historical development.

Chapter 1: Basic Concepts: Groups, rings, fields, modules, and Galois theory.

Chapter 2: Algebraic Number Fields: Definitions, ring of integers, ideal theory, and unique factorization of ideals.

Chapter 3: Class Field Theory: Ideal class groups, Hilbert class fields, Artin reciprocity law.

Chapter 4: Local Fields:  $p$ -adic numbers, local fields, and local-global principles.

Chapter 5: Cohomology of Groups: Introduction to group cohomology and its applications to class field theory.

Conclusion: Summary of key results and directions for further study.

(Detailed Explanation of each point in the outline, expanding on the points already covered above):

The detailed explanation of each point in the outline is essentially the body of the article that precedes this section. It provides a thorough overview of the contents of each chapter in Neukirch's book, explaining the key concepts and theorems covered. This is already incorporated within the body of the article above.

## FAQs:

1. What is the prerequisite knowledge needed to understand Neukirch's book? A strong background in abstract algebra, including group theory, ring theory, and field theory, is essential. Familiarity with Galois theory is highly recommended.
1. Is Neukirch's book suitable for beginners? No, it is considered a graduate-level text and is not suitable for beginners without a solid foundation in abstract algebra.
1. What makes Neukirch's book stand out from other algebraic number theory texts? Its rigorous yet clear presentation, its comprehensive coverage, and its emphasis on class field theory are key distinguishing features.
1. What are the applications of algebraic number theory? It has wide-ranging applications in cryptography, coding theory, and other areas of mathematics.

1. Is there an easier introduction to algebraic number theory before tackling Neukirch? Yes, there are many introductory texts that provide a gentler introduction to the subject before delving into the depth of Neukirch's book.
1. How long does it typically take to work through Neukirch's book? This depends on the reader's background and pace, but it can take several months or even a year of dedicated study.
1. Are there any online resources to help with understanding Neukirch? While there isn't a single comprehensive online resource, lecture notes and online forums can be helpful supplements.
1. What is the best way to approach studying Neukirch's book? Work through the exercises carefully, and don't hesitate to consult other texts or resources for clarification.
1. Is there a solutions manual available for the exercises in Neukirch's book? No official solutions manual exists, but solutions to some exercises can be found online in various forums and communities.

#### Related Articles:

1. Introduction to Algebraic Number Fields: A beginner-friendly overview of the fundamental concepts.
2. Galois Theory and its Applications: A detailed exploration of Galois theory, essential for understanding algebraic number theory.
3. Ideal Theory in Algebraic Number Fields: A deep dive into the theory of ideals and their role in algebraic number fields.
4. Class Field Theory: An Overview: A less technical overview of the core ideas of class field theory.
5. The Artin Reciprocity Law: A Gentle Introduction: An explanation of this central theorem in a more accessible way.

6. **p-adic Numbers and their Properties:** An exploration of p-adic numbers and their significance in algebraic number theory.
7. **Local Fields and Global Fields:** A comparison of the properties and applications of local and global fields.
8. **Cohomology of Groups in Algebraic Number Theory:** An in-depth look at the role of group cohomology.
9. **Applications of Algebraic Number Theory in Cryptography:** Exploring the practical use of algebraic number theory in securing information.

**neukirch algebraic number theory:** Algebraic Number Theory Jürgen Neukirch, 2013-03-14

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

**neukirch algebraic number theory:** *Algebraic Number Theory* Jürgen Neukirch, 2010-12-15

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

**neukirch algebraic number theory:** *Algebraic Number Theory* Jürgen Neukirch, 1999-05-21

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

**neukirch algebraic number theory:** □□□□ Jürgen Neukirch, 1999 □□□□□□□□□□□□□□

**neukirch algebraic number theory: Quadratic Number Theory** J. L. Lehman, 2019-02-13

Quadratic Number Theory is an introduction to algebraic number theory for readers with a moderate knowledge of elementary number theory and some familiarity with the terminology of abstract algebra. By restricting attention to questions about squares the author achieves the dual goals of making the presentation accessible to undergraduates and reflecting the historical roots of the subject. The representation of integers by quadratic forms is emphasized throughout the text. Lehman introduces an innovative notation for ideals of a quadratic domain that greatly facilitates computation and he uses this to particular effect. The text has an unusual focus on actual computation. This focus, and this notation, serve the author's historical purpose as well; ideals can be seen as number-like objects, as Kummer and Dedekind conceived of them. The notation can be adapted to quadratic forms and provides insight into the connection between quadratic forms and ideals. The computation of class groups and continued fraction representations are featured—the author's notation makes these computations particularly illuminating. Quadratic Number Theory, with its exceptionally clear prose, hundreds of exercises, and historical motivation, would make an excellent textbook for a second undergraduate course in number theory. The clarity of the exposition would also make it a terrific choice for independent reading. It will be exceptionally useful as a fruitful launching pad for undergraduate research projects in algebraic number theory.

**neukirch algebraic number theory:** *Algebraic Number Theory* Jürgen Neukirch, 2012-12-22

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of

Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

**neukirch algebraic number theory: *Cohomology of Number Fields*** Jürgen Neukirch, Alexander Schmidt, Kay Wingberg, 2013-09-26 This second edition is a corrected and extended version of the first. It is a textbook for students, as well as a reference book for the working mathematician, on cohomological topics in number theory. In all it is a virtually complete treatment of a vast array of central topics in algebraic number theory. New material is introduced here on duality theorems for unramified and tamely ramified extensions as well as a careful analysis of 2-extensions of real number fields.

**neukirch algebraic number theory: *Algebraic Number Theory*** A. Fröhlich, Martin J. Taylor, 1991 This book originates from graduate courses given in Cambridge and London. It provides a brisk, thorough treatment of the foundations of algebraic number theory, and builds on that to introduce more advanced ideas. Throughout, the authors emphasise the systematic development of techniques for the explicit calculation of the basic invariants, such as rings of integers, class groups, and units. Moreover they combine, at each stage of development, theory with explicit computations and applications, and provide motivation in terms of classical number-theoretic problems. A number of special topics are included that can be treated at this level but can usually only be found in research monographs or original papers, for instance: module theory of Dedekind domains; tame and wild ramifications; Gauss series and Gauss periods; binary quadratic forms; and Brauer relations. This is the only textbook at this level which combines clean, modern algebraic techniques together with a substantial arithmetic content. It will be indispensable for all practising and would-be algebraic number theorists.

**neukirch algebraic number theory: *A Brief Guide to Algebraic Number Theory*** H. P. F. Swinnerton-Dyer, 2001-02-22 Broad graduate-level account of Algebraic Number Theory, first published in 2001, including exercises, by a world-renowned author.

**neukirch algebraic number theory: *Problems in Algebraic Number Theory*** M. Ram Murty, Jody (Indigo) Esmonde, 2005-09-28 The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved

**neukirch algebraic number theory: *Classical Theory of Algebraic Numbers*** Paulo Ribenboim, 2013-11-11 The exposition of the classical theory of algebraic numbers is clear and thorough, and there is a large number of exercises as well as worked out numerical examples. A careful study of this book will provide a solid background to the learning of more recent topics.

**neukirch algebraic number theory: *Class Field Theory*** J. Neukirch, 2012-12-06 Class field theory, which is so immediately compelling in its main assertions, has, ever since its invention, suffered from the fact that its proofs have required a complicated and, by comparison with the results, rather imper spicuous system of arguments which have tended to jump around all over the place. My earlier presentation of the theory [41] has strengthened me in the belief that a highly elaborate mechanism, such as, for example, cohomology, might not be adequate for a number-theoretical law admitting a very direct formulation, and that the truth of such a law must be susceptible to a far more immediate insight. I was determined to write the present, new account of class field theory by the discovery that, in fact, both the local and the global reciprocity laws may be subsumed under a purely group theoretical principle, admitting an entirely elementary description. This description makes possible a new foundation for the entire theory. The rapid advance to the main theorems of class field theory which results from this approach has made it possible to include in this volume the most important consequences and elaborations, and further related theories, with the exception of the cohomology version which I have this time excluded. This remains a significant variant, rich in application, but its principal results should be directly obtained from the material treated here.

**neukirch algebraic number theory: *Class Field Theory*** Jürgen Neukirch, 2013-04-08 The

present manuscript is an improved edition of a text that first appeared under the same title in Bonner Mathematische Schriften, no.26, and originated from a series of lectures given by the author in 1965/66 in Wolfgang Krull's seminar in Bonn. Its main goal is to provide the reader, acquainted with the basics of algebraic number theory, a quick and immediate access to class field theory. This script consists of three parts, the first of which discusses the cohomology of finite groups. The second part discusses local class field theory, and the third part concerns the class field theory of finite algebraic number fields.

**neukirch algebraic number theory: *Algebraic Number Theory and Fermat's Last Theorem*** Ian Stewart, David Tall, 2001-12-12 First published in 1979 and written by two distinguished mathematicians with a special gift for exposition, this book is now available in a completely revised third edition. It reflects the exciting developments in number theory during the past two decades that culminated in the proof of Fermat's Last Theorem. Intended as an upper level textbook, it

**neukirch algebraic number theory: *Basic Number Theory*** Andre Weil, 1995-02-15 From the reviews: L.R. Shafarevich showed me the first edition [...] and said that this book will be from now on the book about class field theory. In fact it is by far the most complete treatment of the main theorems of algebraic number theory, including function fields over finite constant fields, that appeared in book form. Zentralblatt MATH

**neukirch algebraic number theory: *Algebraic Number Theory*** Serge Lang, 2013-06-29 This is a second edition of Lang's well-known textbook. It covers all of the basic material of classical algebraic number theory, giving the student the background necessary for the study of further topics in algebraic number theory, such as cyclotomic fields, or modular forms. Lang's books are always of great value for the graduate student and the research mathematician. This updated edition of Algebraic number theory is no exception.—MATHEMATICAL REVIEWS

**neukirch algebraic number theory: *Commutative Algebra*** David Eisenbud, 2013-12-01 This is a comprehensive review of commutative algebra, from localization and primary decomposition through dimension theory, homological methods, free resolutions and duality, emphasizing the origins of the ideas and their connections with other parts of mathematics. The book gives a concise treatment of Grobner basis theory and the constructive methods in commutative algebra and algebraic geometry that flow from it. Many exercises included.

**neukirch algebraic number theory: *Number Fields*** Daniel A. Marcus, 2018-07-05 Requiring no more than a basic knowledge of abstract algebra, this text presents the mathematics of number fields in a straightforward, pedestrian manner. It therefore avoids local methods and presents proofs in a way that highlights the important parts of the arguments. Readers are assumed to be able to fill in the details, which in many places are left as exercises.

**neukirch algebraic number theory: *Quadratic Forms Over Semilocal Rings*** R. Baeza, 2006-11-22

**neukirch algebraic number theory: *Algebraic Number Theory*** Ian Stewart, 1979-05-31 The title of this book may be read in two ways. One is 'algebraic number-theory', that is, the theory of numbers viewed algebraically; the other, 'algebraic-number theory', the study of algebraic numbers. Both readings are compatible with our aims, and both are perhaps misleading. Misleading, because a proper coverage of either topic would require more space than is available, and demand more of the reader than we wish to; compatible, because our aim is to illustrate how some of the basic notions of the theory of algebraic numbers may be applied to problems in number theory. Algebra is an easy subject to compartmentalize, with topics such as 'groups', 'rings' or 'modules' being taught in comparative isolation. Many students view it this way. While it would be easy to exaggerate this tendency, it is not an especially desirable one. The leading mathematicians of the nineteenth and early twentieth centuries developed and used most of the basic results and techniques of linear algebra for perhaps a hundred years, without ever defining an abstract vector space: nor is there anything to suggest that they suffered thereby. This historical fact may indicate that abstraction is not always as necessary as one commonly imagines; on the other hand the axiomatization of mathematics has led to enormous organizational and conceptual gains.

**neukirch algebraic number theory: Number Theory and Algebraic Geometry** Miles Reid, Alexei Skorobogatov, 2003 This volume honors Sir Peter Swinnerton-Dyer's mathematical career spanning more than 60 years' of amazing creativity in number theory and algebraic geometry.

**neukirch algebraic number theory: Number Theory** , 1986-05-05 This book is written for the student in mathematics. Its goal is to give a view of the theory of numbers, of the problems with which this theory deals, and of the methods that are used. We have avoided that style which gives a systematic development of the apparatus and have used instead a freer style, in which the problems and the methods of solution are closely interwoven. We start from concrete problems in number theory. General theories arise as tools for solving these problems. As a rule, these theories are developed sufficiently far so that the reader can see for himself their strength and beauty, and so that he learns to apply them. Most of the questions that are examined in this book are connected with the theory of diophantine equations - that is, with the theory of the solutions in integers of equations in several variables. However, we also consider questions of other types; for example, we derive the theorem of Dirichlet on prime numbers in arithmetic progressions and investigate the growth of the number of solutions of congruences.

**neukirch algebraic number theory: Number Theory II** A. N. Parshin, Игорь Ростиславович Шафаревич, 1992 Volume 62 of the Encyclopedia presents the main structures and results of algebraic number theory with emphasis on algebraic number fields and class field theory. Written for the nonspecialist, the author assumes a general understanding of modern algebra and elementary number theory. Only the general properties of algebraic number fields and relate.

**neukirch algebraic number theory: Algebraic Number Theory** Robert L. Long, 1977

**neukirch algebraic number theory: Number Theory** W.A. Coppel, 2006-02-02 This two-volume book is a modern introduction to the theory of numbers, emphasizing its connections with other branches of mathematics. Part A is accessible to first-year undergraduates and deals with elementary number theory. Part B is more advanced and gives the reader an idea of the scope of mathematics today. The connecting theme is the theory of numbers. By exploring its many connections with other branches a broad picture is obtained. The book contains a treasury of proofs, several of which are gems seldom seen in number theory books.

**neukirch algebraic number theory: The Theory of Algebraic Numbers: Second Edition** Harry Pollard, Harold G. Diamond , 1975-12-31 This monograph makes available, in English, the elementary parts of classical algebraic number theory. This second edition follows closely the plan and style of the first edition. The principal changes are the correction of misprints, the expansion or simplification of some arguments, and the omission of the final chapter on units in order to make way for the introduction of some two hundred problems.

**neukirch algebraic number theory: The Geometry of Schemes** David Eisenbud, Joe Harris, 2006-04-06 Grothendieck's beautiful theory of schemes permeates modern algebraic geometry and underlies its applications to number theory, physics, and applied mathematics. This simple account of that theory emphasizes and explains the universal geometric concepts behind the definitions. In the book, concepts are illustrated with fundamental examples, and explicit calculations show how the constructions of scheme theory are carried out in practice.

**neukirch algebraic number theory: Primes of the Form  $X^2 + Ny^2$**  David A. Cox, 1989 Modern number theory began with the work of Euler and Gauss to understand and extend the many unsolved questions left behind by Fermat. In the course of their investigations, they uncovered new phenomena in need of explanation, which over time led to the discovery of class field theory and its intimate connection with complex multiplication. While most texts concentrate on only the elementary or advanced aspects of this story, *Primes of the Form  $x^2 + ny^2$*  begins with Fermat and explains how his work ultimately gave birth to quadratic reciprocity and the genus theory of quadratic forms. Further, the book shows how the results of Euler and Gauss can be fully understood only in the context of class field theory. Finally, in order to bring class field theory down to earth, the book explores some of the magnificent formulas of complex multiplication. The central theme of the book is the story of which primes  $p$  can be expressed in the form  $x^2 + ny^2$ . An incomplete answer is

given using quadratic forms. A better though abstract answer comes from class field theory, and finally, a concrete answer is provided by complex multiplication. Along the way, the reader is introduced to some wonderful number theory. Numerous exercises and examples are included. The book is written to be enjoyed by readers with modest mathematical backgrounds. Chapter 1 uses basic number theory and abstract algebra, while chapters 2 and 3 require Galois theory and complex analysis, respectively.

**neukirch algebraic number theory: *Algebraic Theory of Numbers*** Pierre Samuel, 2008  
Algebraic number theory introduces students to new algebraic notions as well as related concepts: groups, rings, fields, ideals, quotient rings, and quotient fields. This text covers the basics, from divisibility theory in principal ideal domains to the unit theorem, finiteness of the class number, and Hilbert ramification theory. 1970 edition.

**neukirch algebraic number theory: *Groups as Galois Groups*** Helmut Völklein, 1996-08-13  
Develops the mathematical background and recent results on the Inverse Galois Problem.

**neukirch algebraic number theory: *Arithmetic Duality Theorems*** J. S. Milne, 1986 Here, published for the first time, are the complete proofs of the fundamental arithmetic duality theorems that have come to play an increasingly important role in number theory and arithmetic geometry. The text covers these theorems in Galois cohomology, étale cohomology, and flat cohomology and addresses applications in the above areas. The writing is expository and the book will serve as an invaluable reference text as well as an excellent introduction to the subject.

**neukirch algebraic number theory: *Lectures on the Theory of Algebraic Numbers*** E. T. Hecke, 2013-03-09 . . . if one wants to make progress in mathematics one should study the masters not the pupils. N. H. Abel Hecke was certainly one of the masters, and in fact, the study of Hecke L series and Hecke operators has permanently embedded his name in the fabric of number theory. It is a rare occurrence when a master writes a basic book, and Hecke's *Lectures on the Theory of Algebraic Numbers* has become a classic. To quote another master, Andre Weil: To improve upon Hecke, in a treatment along classical lines of the theory of algebraic numbers, would be a futile and impossible task. We have tried to remain as close as possible to the original text in preserving Hecke's rich, informal style of exposition. In a very few instances we have substituted modern terminology for Hecke's, e. g. , torsion free group for pure group. One problem for a student is the lack of exercises in the book. However, given the large number of texts available in algebraic number theory, this is not a serious drawback. In particular we recommend *Number Fields* by D. A. Marcus (Springer-Verlag) as a particularly rich source. We would like to thank James M. Vaughn Jr. and the Vaughn Foundation Fund for their encouragement and generous support of Jay R. Goldman without which this translation would never have appeared. Minneapolis George U. Brauer July 1981 Jay R.

**neukirch algebraic number theory: *Field Arithmetic*** Michael D. Fried, Moshe Jarden, 2005  
Field Arithmetic explores Diophantine fields through their absolute Galois groups. This largely self-contained treatment starts with techniques from algebraic geometry, number theory, and profinite groups. Graduate students can effectively learn generalizations of finite field ideas. We use Haar measure on the absolute Galois group to replace counting arguments. New Chebotarev density variants interpret diophantine properties. Here we have the only complete treatment of Galois stratifications, used by Denef and Loeser, et al, to study Chow motives of Diophantine statements. Progress from the first edition starts by characterizing the finite-field like  $P(\text{seudo})A(l\text{gebraically})C(l\text{osed})$  fields. We once believed PAC fields were rare. Now we know they include valuable Galois extensions of the rationals that present its absolute Galois group through known groups. PAC fields have projective absolute Galois group. Those that are Hilbertian are characterized by this group being pro-free. These last decade results are tools for studying fields by their relation to those with projective absolute group. There are still mysterious problems to guide a new generation: Is the solvable closure of the rationals PAC; and do projective Hilbertian fields have pro-free absolute Galois group (includes Shafarevich's conjecture)?

**neukirch algebraic number theory: *An Invitation to Arithmetic Geometry*** Dino Lorenzini,

2021-12-23 Extremely carefully written, masterfully thought out, and skillfully arranged introduction ... to the arithmetic of algebraic curves, on the one hand, and to the algebro-geometric aspects of number theory, on the other hand. ... an excellent guide for beginners in arithmetic geometry, just as an interesting reference and methodical inspiration for teachers of the subject ... a highly welcome addition to the existing literature. —Zentralblatt MATH The interaction between number theory and algebraic geometry has been especially fruitful. In this volume, the author gives a unified presentation of some of the basic tools and concepts in number theory, commutative algebra, and algebraic geometry, and for the first time in a book at this level, brings out the deep analogies between them. The geometric viewpoint is stressed throughout the book. Extensive examples are given to illustrate each new concept, and many interesting exercises are given at the end of each chapter. Most of the important results in the one-dimensional case are proved, including Bombieri's proof of the Riemann Hypothesis for curves over a finite field. While the book is not intended to be an introduction to schemes, the author indicates how many of the geometric notions introduced in the book relate to schemes, which will aid the reader who goes to the next level of this rich subject.

**neukirch algebraic number theory: Number Theory 1** Kazuya Kato, Nobushige Kurokawa, Takeshi Saitō, 2000 This is the English translation of the original Japanese book. In this volume, Fermat's Dream, core theories in modern number theory are introduced. Developments are given in elliptic curves,  $p$ -adic numbers, the  $\zeta$ -function, and the number fields. This work presents an elegant perspective on the wonder of numbers. Number Theory 2 on class field theory, and Number Theory 3 on Iwasawa theory and the theory of modular forms, are forthcoming in the series.

**neukirch algebraic number theory: A Gentle Course in Local Class Field Theory** Pierre Guillot, 2018-11 A self-contained exposition of local class field theory for students in advanced algebra.

**neukirch algebraic number theory: Basic Number Theory**. Andre Weil, 2013-12-14 Itpzf}JlOV, li~oxov uoq>ZUJlCJ. 7:WV Al(JX., llpoj1. AE(Jj1. The first part of this volume is based on a course taught at Princeton University in 1961-62; at that time, an excellent set of notes was prepared by David Cantor, and it was originally my intention to make these notes available to the mathematical public with only quite minor changes. Then, among some old papers of mine, I accidentally came across a long-forgotten manuscript by Chevalley, of pre-war vintage (forgotten, that is to say, both by me and by its author) which, to my taste at least, seemed to have aged very well. It contained a brief but essentially complete account of the main features of class field theory, both local and global; and it soon became obvious that the usefulness of the intended volume would be greatly enhanced if I included such a treatment of this topic. It had to be expanded, in accordance with my own plans, but its outline could be preserved without much change. In fact, I have adhered to it rather closely at some critical points.

**neukirch algebraic number theory: Rational Points on Elliptic Curves** Joseph H. Silverman, John Tate, 2013-04-17 The theory of elliptic curves involves a blend of algebra, geometry, analysis, and number theory. This book stresses this interplay as it develops the basic theory, providing an opportunity for readers to appreciate the unity of modern mathematics. The book's accessibility, the informal writing style, and a wealth of exercises make it an ideal introduction for those interested in learning about Diophantine equations and arithmetic geometry.

**neukirch algebraic number theory: Local Fields** Jean-Pierre Serre, 2013-06-29 The goal of this book is to present local class field theory from the cohomological point of view, following the method inaugurated by Hochschild and developed by Artin-Tate. This theory is about extensions—primarily abelian—of local (i.e., complete for a discrete valuation) fields with finite residue field. For example, such fields are obtained by completing an algebraic number field; that is one of the aspects of localisation. The chapters are grouped in parts. There are three preliminary parts: the first two on the general theory of local fields, the third on group cohomology. Local class field theory, strictly speaking, does not appear until the fourth part. Here is a more precise outline of the contents of these four parts: The first contains basic definitions and results on discrete valuation rings, Dedekind domains (which are their globalisation) and the completion process. The

prerequisite for this part is a knowledge of elementary notions of algebra and topology, which may be found for instance in Bourbaki. The second part is concerned with ramification phenomena (different, discriminant, ramification groups, Artin representation). Just as in the first part, no assumptions are made here about the residue fields. It is in this setting that the norm map is studied; I have expressed the results in terms of additive polynomials and of multiplicative polynomials, since using the language of algebraic geometry would have led me too far astray.

**neukirch algebraic number theory: An Introduction to the Langlands Program** Joseph Bernstein, Stephen Gelbart, 2013-12-11 This book presents a broad, user-friendly introduction to the Langlands program, that is, the theory of automorphic forms and its connection with the theory of L-functions and other fields of mathematics. Each of the twelve chapters focuses on a particular topic devoted to special cases of the program. The book is suitable for graduate students and researchers.

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