number theory and algebra

Number Theory and Algebra: A Deep Dive into the Intertwined Worlds of Numbers and Symbols

Introduction:

Are you fascinated by the elegance of mathematical structures? Do you find yourself pondering the hidden relationships between numbers and the power of algebraic manipulation? Then you've come to the right place! This comprehensive guide delves into the fascinating and interconnected fields of number theory and algebra, exploring their fundamental concepts, key theorems, and surprising applications. We’ll journey from the basic properties of integers to the abstract beauty of algebraic structures, unveiling the intricate dance between numbers and symbols that shapes our understanding of mathematics and the world around us. Prepare to be amazed by the power and beauty of these fundamental mathematical disciplines!

1. The Foundations of Number Theory: Unraveling the Secrets of Integers

Number theory, often dubbed "the queen of mathematics," is the study of integers (whole numbers) and their properties. It explores concepts like:

Divisibility: Understanding when one integer divides another without leaving a remainder is foundational. We examine prime numbers (divisible only by 1 and themselves), composite numbers (having more than two divisors), and the fundamental theorem of arithmetic (unique prime factorization). This seemingly simple concept underpins much of cryptography and modern computer security.

Congruences: Modular arithmetic, focusing on remainders after division, introduces congruences. This leads to powerful tools like Fermat's Little Theorem and Chinese Remainder Theorem, with applications ranging from cryptography to scheduling and calendar calculations.

Diophantine Equations: These are equations where we seek integer solutions. Famous examples include Fermat's Last Theorem (proven only recently), and Pell's equation, which has fascinating connections to continued fractions and the approximation of irrational numbers. The search for integer solutions often leads to deep insights into the structure of numbers themselves.

Prime Number Distribution: The seemingly random distribution of prime numbers has captivated mathematicians for centuries. The Prime Number Theorem provides an approximation of the number of primes less than a given number, and the Riemann Hypothesis (one of the Millennium Prize Problems) delves even deeper into the mysterious distribution of these fundamental building blocks of arithmetic.

2. The Elegance of Algebra: Exploring Structures and Transformations

Algebra, in its broadest sense, is the study of mathematical structures and their properties using symbols and operations. It encompasses:

Basic Algebraic Structures: This includes groups, rings, and fields. A group is a set with a binary operation satisfying certain properties (closure, associativity, identity, and inverses). Rings add another operation (usually multiplication), while fields are rings where every non-zero element has a multiplicative inverse. These abstract structures provide a framework for understanding various mathematical systems.

Polynomial Algebra: Polynomials are expressions involving variables and coefficients. We explore their properties, such as factoring, finding roots (solutions), and applying them to solve equations and model real-world phenomena. The fundamental theorem of algebra states that a polynomial of degree n has exactly n complex roots (counting multiplicity).

Linear Algebra: This branch focuses on vector spaces, matrices, and linear transformations. It has profound applications in computer graphics, machine learning, and quantum mechanics. Eigenvalues and eigenvectors, for instance, reveal crucial information about the structure of linear transformations.

Abstract Algebra: This extends the concepts of groups, rings, and fields to more abstract settings, leading to powerful generalizations and deeper understanding of mathematical structures. Galois theory, for instance, uses group theory to study the solvability of polynomial equations by radicals.

3. The Interplay Between Number Theory and Algebra: A Symbiotic Relationship

Number theory and algebra are not isolated disciplines; they are deeply intertwined. Algebraic techniques are frequently used to solve problems in number theory, and number-theoretic concepts often illuminate algebraic structures. For example:

Using algebraic tools to study Diophantine equations: Advanced algebraic techniques, such as those from algebraic geometry, are crucial in tackling complex Diophantine equations.

Applying group theory to understand the structure of number systems: Group theory provides a powerful framework for analyzing the structure of various number systems, such as the integers modulo n.

Utilizing ring theory to explore properties of polynomials: Ring theory provides the necessary tools to understand the factorization of polynomials and their properties over different fields.

4. Applications of Number Theory and Algebra: From Cryptography to Quantum Computing

The applications of number theory and algebra are vast and far-reaching:

Cryptography: Public-key cryptography, which underpins secure online communication, relies heavily on number theory concepts like prime factorization and modular arithmetic.

Coding Theory: Error-correcting codes, essential for reliable data transmission, draw upon algebraic structures and number-theoretic properties.

Computer Science: Algorithms in computer science often leverage algebraic structures and number-theoretic principles for efficient computation.

Physics: Algebraic structures play a critical role in quantum mechanics and other areas of theoretical physics.

A Sample Textbook Outline: "Exploring Number Theory and Algebra"

Introduction: A brief overview of number theory and algebra, their historical development, and their interconnections.

Chapter 1: Fundamentals of Number Theory: Divisibility, prime numbers, the fundamental theorem of arithmetic, congruences, and modular arithmetic.

Chapter 2: Advanced Number Theory: Diophantine equations, quadratic reciprocity, and the distribution of prime numbers.

Chapter 3: Introduction to Abstract Algebra: Groups, subgroups, homomorphisms, and group actions.

Chapter 4: Rings and Fields: Ring theory, field extensions, and polynomial rings.

Chapter 5: Linear Algebra: Vector spaces, linear transformations, eigenvalues, and eigenvectors.

Chapter 6: Applications: Cryptography, coding theory, and other applications of number theory and algebra.

Conclusion: Summary of key concepts and a look towards advanced topics.

(Note: Each chapter would contain numerous sections, theorems, examples, and exercises.)

Detailed Explanation of the Textbook Outline Points:

Each point in the textbook outline would be elaborated upon extensively in the corresponding chapter. For example, Chapter 1 on "Fundamentals of Number Theory" would define divisibility, prove the fundamental theorem of arithmetic, introduce concepts like greatest common divisor (GCD) and least common multiple (LCM), delve into Euclid's algorithm for finding GCDs, and explain modular arithmetic and its applications. Similarly, Chapter 4 on "Rings and Fields" would introduce the axioms for rings and fields, explore examples of various rings and fields (like integers, polynomials, rational numbers, etc.), delve into ideals and quotient rings, and introduce concepts like field extensions. The concluding chapter would provide a comprehensive summary of all the concepts learned and would point towards more advanced topics in both number theory and algebra.

FAQs

1. What is the difference between number theory and algebra? Number theory focuses on the properties of integers, while algebra studies mathematical structures using symbols and operations.

1. What are some real-world applications of number theory? Cryptography, coding theory, and computer science algorithms all rely on number theory.

1. What are some real-world applications of algebra? Linear algebra is crucial in computer graphics, machine learning, and quantum mechanics.

1. Is number theory harder than algebra? The relative difficulty depends on the specific topics and the individual's strengths. Both fields have challenging aspects.

1. What are some famous unsolved problems in number theory? The Riemann Hypothesis and Goldbach's conjecture are notable examples.

1. What are some important theorems in algebra? The fundamental theorem of algebra and various theorems from group theory and ring theory are significant.

1. How are number theory and algebra related? They are deeply interconnected; algebraic techniques are used to solve number theory problems, and number-theoretic concepts often illuminate algebraic structures.

1. What mathematical background is needed to study number theory and algebra? A solid foundation in high school mathematics (including algebra and geometry) is typically required.

1. Where can I find more information about number theory and algebra? Numerous textbooks, online courses, and research papers are available on these subjects.

Related Articles:

1. Introduction to Modular Arithmetic: Explores the basic concepts and properties of modular arithmetic, including congruences and their applications.

1. Prime Numbers and the Riemann Hypothesis: Delves into the distribution of prime numbers and the significance of the Riemann Hypothesis.

1. Diophantine Equations: A Beginner's Guide: Introduces various types of Diophantine equations and methods for solving them.

1. Group Theory and Its Applications: Explains the fundamental concepts of

group theory and its applications in different fields.

1. Ring Theory: An Overview: Provides a comprehensive overview of ring theory, including its axioms, key concepts, and examples.

1. Field Extensions and Galois Theory: Introduces the concepts of field extensions and Galois theory, focusing on their connection to the solvability of polynomial equations.

1. Linear Algebra for Beginners: A gentle introduction to the basic concepts of linear algebra, including vectors, matrices, and linear transformations.

1. Public-Key Cryptography and Number Theory: Explores the connection between public-key cryptography and number theory, showcasing the use of prime numbers and modular arithmetic.

1. Error-Correcting Codes and Algebra: Explains how algebraic structures are used in the design and implementation of error-correcting codes.

number theory and algebra: Algebra and Number Theory Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya Subbotin, 2011-07-15 Explore the main algebraic structures and number systems that play a central role across the field of mathematics Algebra and number theory are two powerful branches of modern mathematics at the forefront of current mathematical research, and each plays an increasingly significant role in different branches of mathematics, from geometry and topology to computing and communications. Based on the authors' extensive experience within the field, Algebra and Number Theory has an innovative approach that integrates three disciplines—linear algebra, abstract algebra, and number theory—into one comprehensive and fluid presentation, facilitating a deeper understanding of the topic and improving readers' retention of the main concepts. The book begins with an introduction to the elements of set theory. Next, the authors discuss matrices, determinants, and elements of field theory, including preliminary information related to integers and complex numbers. Subsequent chapters explore key ideas relating to linear algebra such as vector spaces, linear mapping, and bilinear forms. The book explores the development of the main ideas of algebraic structures and concludes with applications of algebraic ideas to number theory. Interesting applications are provided throughout to demonstrate the relevance of the discussed concepts. In addition, chapter exercises allow readers to test their comprehension of the presented material. Algebra and Number Theory is an excellent book for courses on linear algebra, abstract algebra, and number theory at the upper-undergraduate level. It is also a valuable reference for researchers working in different fields of mathematics, computer science, and engineering as well as for individuals preparing for a career in mathematics education.

number theory and algebra: Algebraic Number Theory Edwin Weiss, 2012-01-27 Ideal either for classroom use or as exercises for mathematically minded individuals, this text introduces elementary valuation theory, extension of valuations, local and ordinary arithmetic fields, and global, quadratic, and cyclotomic fields.

number theory and algebra: Algebraic Number Theory Serge Lang, 2013-06-29 This is a second edition of Lang's well-known textbook. It covers all of the basic material of classical algebraic number theory, giving the student the background necessary for the study of further topics in algebraic number theory, such as cyclotomic fields, or modular forms. Lang's books are always of great value for the graduate student and the research mathematician. This updated edition of Algebraic number theory is no exception.—-MATHEMATICAL REVIEWS

number theory and algebra: Fundamentals of Number Theory William J. LeVeque, 2014-01-05 This excellent textbook introduces the basics of number theory, incorporating the language of abstract algebra. A knowledge of such algebraic concepts as group, ring, field, and domain is not assumed, however; all terms are defined and examples are given — making the book self-contained in this respect. The author begins with an introductory chapter on number theory and its early history. Subsequent chapters deal with unique factorization and the GCD, quadratic residues, number-theoretic functions and the distribution of primes, sums of squares, quadratic equations and quadratic fields, diophantine approximation, and more. Included are discussions of topics not always found in introductory texts: factorization and primality of large integers, p-adic numbers, algebraic number fields, Brun's theorem on twin primes, and the transcendence of e, to mention a few. Readers will find a substantial number of well-chosen problems, along with many notes and bibliographical references selected for readability and relevance. Five helpful appendixes — containing such study aids as a factor table, computer-plotted graphs, a table of indices, the Greek alphabet, and a list of symbols — and a bibliography round out this well-written text, which is directed toward undergraduate majors and beginning graduate students in mathematics. No post-calculus prerequisite is assumed. 1977 edition.

number theory and algebra: Lectures on the Theory of Algebraic Numbers E. T. Hecke, 2013-03-09 . . . if one wants to make progress in mathematics one should study the masters not the pupils. N. H. Abel Heeke was certainly one of the masters, and in fact, the study of Heeke L series and Heeke operators has permanently embedded his name in the fabric of number theory. It is a rare occurrence when a master writes a basic book, and Heeke's Lectures on the Theory of Algebraic Numbers has become a classic. To quote another master, Andre Weil: To improve upon Heeke, in a treatment along classical lines of the theory of algebraic numbers, would be a futile and impossible task. We have tried to remain as close as possible to the original text in pre serving Heeke's rich, informal style of exposition. In a very few instances we have substituted modern terminology for Heeke's, e. g. , torsion free group for pure group. One problem for a student is the lack of exercises in the book. However, given the large number of texts available in algebraic number theory, this is not a serious drawback. In particular we recommend Number Fields by D. A. Marcus (Springer-Verlag) as a particularly rich source. We would like to thank James M. Vaughn Jr. and the Vaughn Foundation Fund for their encouragement and generous support of Jay R. Goldman without which this translation would never have appeared. Minneapolis George U. Brauer July 1981 Jay R.

number theory and algebra: Algebraic Number Theory and Fermat's Last Theorem Ian Stewart, David Tall, 2001-12-12 First published in 1979 and written by two distinguished mathematicians with a special gift for exposition, this book is now available in a completely revised third edition. It reflects the exciting developments in number theory during the past two decades that culminated in the proof of Fermat's Last Theorem. Intended as a upper level textbook, it

number theory and algebra: An Introduction to Commutative Algebra and Number Theory Sukumar Das Adhikari, 2001-11 This is an elementary introduction to algebra and number theory. The text begins by a review of groups, rings, and fields. The algebra portion addresses polynomial rings, UFD, PID, and Euclidean domains, field extensions, modules, and Dedckind domains. The number theory portion reviews elementary congruence, quadratic reciprocity, algebraic number fields, and a glimpse into the various aspects of that subject. This book could be used as a one semester course in graduate mathematics.

number theory and algebra: Algebraic Number Theory Frazer Jarvis, 2014-06-23 This undergraduate textbook provides an approachable and thorough introduction to the topic of algebraic number theory, taking the reader from unique factorisation in the integers through to the modern-day number field sieve. The first few chapters consider the importance of arithmetic in fields larger than the rational numbers. Whilst some results generalise well, the unique factorisation of the integers in these more general number fields often fail. Algebraic number theory aims to overcome this problem. Most examples are taken from quadratic fields, for which calculations are easy to perform. The middle section considers more general theory and results for number fields, and the book concludes with some topics which are more likely to be suitable for advanced students, namely, the analytic class number formula and the number field sieve. This is the first time that the number field sieve has been considered in a textbook at this level.

number theory and algebra: *A Course in Computational Algebraic Number Theory* Henri Cohen, 2013-04-17 A description of 148 algorithms fundamental to number-theoretic computations, in particular for computations related to algebraic number theory, elliptic curves, primality testing and factoring. The first seven chapters guide readers to the heart of current research in computational algebraic number theory, including recent algorithms for computing class groups and units, as well as elliptic curve computations, while the last three chapters survey factoring and primality testing methods, including a detailed description of the number field sieve algorithm. The whole is rounded off with a description of available computer packages and some useful tables, backed by numerous exercises. Written by an authority in the field, and one with great practical and teaching experience, this is certain to become the standard and indispensable reference on the subject.

number theory and algebra: Algebraic Number Theory Jürgen Neukirch, 2013-03-14 This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

number theory and algebra: Number Theory and Its History Oystein Ore, 2012-07-06 Unusually clear, accessible introduction covers counting, properties of numbers, prime numbers, Aliquot parts, Diophantine problems, congruences, much more. Bibliography.

number theory and algebra: A Brief Guide to Algebraic Number Theory H. P. F. Swinnerton-Dyer, 2001-02-22 Broad graduate-level account of Algebraic Number Theory, first published in 2001, including exercises, by a world-renowned author.

number theory and algebra: Introduction to Cyclotomic Fields Lawrence C. Washington, 2012-12-06 This text on a central area of number theory covers p-adic L-functions, class numbers, cyclotomic units, Fermat’s Last Theorem, and Iwasawa’s theory of Z_p-extensions. This edition contains a new chapter on the work of Thaine, Kolyvagin, and Rubin, including a proof of the Main Conjecture, as well as a chapter on other recent developments, such as primality testing via Jacobi sums and Sinnott’s proof of the vanishing of Iwasawa’s f-invariant.

number theory and algebra: *An Adventurer's Guide to Number Theory* Richard Friedberg, 2012-07-06 This witty introduction to number theory deals with the properties of numbers and numbers as abstract concepts. Topics include primes, divisibility, quadratic forms, and related theorems.

number theory and algebra: Algebraic Number Theory Ian Stewart, 1979-05-31 The title of this book may be read in two ways. One is 'algebraic number-theory', that is, the theory of numbers viewed algebraically; the other, 'algebraic-number theory', the study of algebraic numbers. Both readings are compatible with our aims, and both are perhaps misleading. Misleading, because a proper coverage of either topic would require more space than is available, and demand more of the reader than we wish to; compatible, because our aim is to illustrate how some of the basic notions of the theory of algebraic numbers may be applied to problems in number theory. Algebra is an easy subject to compartmentalize, with topics such as 'groups', 'rings' or 'modules' being taught in comparative isolation. Many students view it this way. While it would be easy to exaggerate this tendency, it is not an especially desirable one. The leading mathematicians of the nineteenth and early twentieth centuries developed and used most of the basic results and techniques of linear algebra for perhaps a hundred years, without ever defining an abstract vector space: nor is there anything to suggest that they suf fered thereby. This historical fact may indicate that abstrac tion is not always as necessary as one commonly imagines; on the other hand the axiomatization of mathematics has led to enormous organizational and conceptual gains.

number theory and algebra: Algebraic Number Theory A. Fröhlich, Martin J. Taylor, 1991 This book originates from graduate courses given in Cambridge and London. It provides a brisk, thorough treatment of the foundations of algebraic number theory, and builds on that to introduce more advanced ideas. Throughout, the authors emphasise the systematic development of techniques for the explicit calculation of the basic invariants, such as rings of integers, class groups, and units. Moreover they combine, at each stage of development, theory with explicit computations and applications, and provide motivation in terms of classical number-theoretic problems. A number of special topics are included that can be treated at this level but can usually only be found in research monographs or original papers, for instance: module theory of Dedekind domains; tame and wild ramifications; Gauss series and Gauss periods; binary quadratic forms; and Brauer relations. This is the only textbook at this level which combines clean, modern algebraic techniques together with a substantial arithmetic content. It will be indispensable for all practising and would-be algebraic number theorists.

number theory and algebra: A Classical Introduction to Modern Number Theory K. Ireland, M. Rosen, 2013-03-09 This book is a revised and greatly expanded version of our book Elements of Number Theory published in 1972. As with the first book the primary audience we envisage consists of upper level undergraduate mathematics majors and graduate students. We have assumed some familiarity with the material in a standard undergraduate course in abstract algebra. A large portion of Chapters 1-11 can be read even without such background with the aid of a small amount of supplementary reading. The later chapters assume some knowledge of Galois theory, and in Chapters 16 and 18 an acquaintance with the theory of complex variables is necessary. Number theory is an ancient subject and its content is vast. Any intro ductory book must, of necessity, make a very limited selection from the fascinat ing array of possible topics. Our focus is on topics which point in the direction of algebraic number theory and arithmetic algebraic geometry. By a careful selection of subject matter we have found it possible to exposit some rather advanced material without requiring very much in the way oftechnical background. Most of this material is classical in the sense that is was dis covered during the nineteenth century and earlier, but it is also modern because it is intimately related to important research going on at the present time.

number theory and algebra: Algebraic Number Theory H. Koch, 2012-12-06 From the reviews: ... The author succeeded in an excellent way to describe the various points of view under which Class Field Theory can be seen. ... In any case the author succeeded to write a very readable book on these difficult themes. Monatshefte fuer Mathematik, 1994 ... Number theory is not easy and quite technical at several places, as the author is able to show in his technically good exposition. The amount of difficult material well exposed gives a survey of quite a lot of good solid classical number theory... Conclusion: for people not already familiar with this field this book is not so easy to read, but for the specialist in number theory this is a useful description of (classical) algebraic number theory. Medelingen van het wiskundig genootschap, 1995

number theory and algebra: Number Theory George E. Andrews, 2012-04-30 Undergraduate text uses combinatorial approach to accommodate both math majors and liberal arts students. Covers the basics of number theory, offers an outstanding introduction to partitions, plus chapters on multiplicativity-divisibility, quadratic congruences, additivity, and more.

number theory and algebra: *Excursions in Number Theory* Charles Stanley Ogilvy, John Timothy Anderson, 1988-01-01 Challenging, accessible mathematical adventures involving prime numbers, number patterns, irrationals and iterations, calculating prodigies, and more. No special training is needed, just high school mathematics and an inquisitive mind. A splendidly written, well selected and presented collection. I recommend the book unreservedly to all readers. — Martin Gardner.

number theory and algebra: Problems in Algebraic Number Theory M. Ram Murty, Jody (Indigo) Esmonde, 2005-09-28 The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved

number theory and algebra: Algebraic Number Theory John Coates, 1989

number theory and algebra: A Course in Number Theory H. E. Rose, 1995 This textbook covers the main topics in number theory as taught in universities throughout the world. Number theory deals mainly with properties of integers and rational numbers; it is not an organized theory in the usual sense but a vast collection of individual topics and results, with some coherent sub-theories and a long list of unsolved problems. This book excludes topics relying heavily on complex analysis and advanced algebraic number theory. The increased use of computers in number theory is reflected in many sections (with much greater emphasis in this edition). Some results of a more advanced nature are also given, including the Gelfond-Schneider theorem, the prime number theorem, and the Mordell-Weil theorem. The latest work on Fermat's last theorem is also briefly discussed. Each chapter ends with a collection of problems; hints or sketch solutions are given at the end of the book, together with various useful tables.

number theory and algebra: Commutative Algebra David Eisenbud, 2013-12-01 This is a comprehensive review of commutative algebra, from localization and primary decomposition through dimension theory, homological methods, free resolutions and duality, emphasizing the origins of the ideas and their connections with other parts of mathematics. The book gives a concise treatment of Grobner basis theory and the constructive methods in commutative algebra and algebraic geometry that flow from it. Many exercises included.

number theory and algebra: *A Computational Introduction to Number Theory and Algebra* Victor Shoup, 2009 An introductory graduate-level text emphasizing algorithms and applications. This second edition includes over 200 new exercises and examples.

number theory and algebra: Number Theory Róbert Freud, Edit Gyarmati, 2020-10-08 Number Theory is a newly translated and revised edition of the most popular introductory textbook on the subject in Hungary. The book covers the usual topics of introductory number theory: divisibility, primes, Diophantine equations, arithmetic functions, and so on. It also introduces several more advanced topics including congruences of higher degree, algebraic number theory, combinatorial number theory, primality testing, and cryptography. The development is carefully laid out with ample illustrative examples and a treasure trove of beautiful and challenging problems. The exposition is both clear and precise. The book is suitable for both graduate and undergraduate courses with enough material to fill two or more semesters and could be used as a source for independent study and capstone projects. Freud and Gyarmati are well-known mathematicians and mathematical educators in Hungary, and the Hungarian version of this book is legendary there. The authors' personal pedagogical style as a facet of the rich Hungarian tradition shines clearly through. It will inspire and exhilarate readers.

number theory and algebra: Equations and Inequalities Jiri Herman, Radan Kucera, Jaromir Simsa, 2012-12-06 A look at solving problems in three areas of classical elementary mathematics: equations and systems of equations of various kinds, algebraic inequalities, and elementary number theory, in particular divisibility and diophantine equations. In each topic, brief theoretical discussions are followed by carefully worked out examples of increasing difficulty, and by exercises which range from routine to rather more challenging problems. While it emphasizes some methods that are not usually covered in beginning university courses, the book nevertheless teaches techniques and skills which are useful beyond the specific topics covered here. With approximately 330 examples and 760 exercises.

number theory and algebra: Algebraic Theory of Numbers Pierre Samuel, 2008 Algebraic number theory introduces students to new algebraic notions as well as related concepts: groups, rings, fields, ideals, quotient rings, and quotient fields. This text covers the basics, from divisibility theory in principal ideal domains to the unit theorem, finiteness of the class number, and Hilbert ramification theory. 1970 edition.

number theory and algebra: Undergraduate Commutative Algebra Miles Reid, 1995-11-30 Commutative algebra is at the crossroads of algebra, number theory and algebraic geometry. This textbook is affordable and clearly illustrated, and is intended for advanced undergraduate or beginning graduate students with some previous experience of rings and fields. Alongside standard algebraic notions such as generators of modules and the ascending chain condition, the book develops in detail the geometric view of a commutative ring as the ring of functions on a space. The starting point is the Nullstellensatz, which provides a close link between the geometry of a variety V and the algebra of its coordinate ring A=k[V]; however, many of the geometric ideas arising from varieties apply also to fairly general rings. The final chapter relates the material of the book to more advanced topics in commutative algebra and algebraic geometry. It includes an account of some famous 'pathological' examples of Akizuki and Nagata, and a brief but thought-provoking essay on the changing position of abstract algebra in today's world.

number theory and algebra: *Number Theory* W.A. Coppel, 2006-02-02 This two-volume book is a modern introduction to the theory of numbers, emphasizing its connections with other branches of mathematics. Part A is accessible to first-year undergraduates and deals with elementary number theory. Part B is more advanced and gives the reader an idea of the scope of mathematics today. The connecting theme is the theory of numbers. By exploring its many connections with other branches a broad picture is obtained. The book contains a treasury of proofs, several of which are gems seldom seen in number theory books.

number theory and algebra: Algebraic Groups and Number Theory Vladimir Platonov, Andrei Rapinchuk, Rachel Rowen, 1993-12-07 This milestone work on the arithmetic theory of linear algebraic groups is now available in English for the first time. Algebraic Groups and Number Theory provides the first systematic exposition in mathematical literature of the junction of group theory, algebraic geometry, and number theory. The exposition of the topic is built on a synthesis of methods from algebraic geometry, number theory, analysis, and topology, and the result is a systematic overview ofalmost all of the major results of the arithmetic theory of algebraic groups obtained to date.

number theory and algebra: Elements of Number Theory John Stillwell, 2012-11-12 Solutions of equations in integers is the central problem of number theory and is the focus of this book. The amount of material is suitable for a one-semester course. The author has tried to avoid the ad hoc proofs in favor of unifying ideas that work in many situations. There are exercises at the end of almost every section, so that each new idea or proof receives immediate reinforcement.

number theory and algebra: Geometric Methods in Algebra and Number Theory Fedor Bogomolov, Yuri Tschinkel, 2006-06-22 * Contains a selection of articles exploring geometric approaches to problems in algebra, algebraic geometry and number theory * The collection gives a representative sample of problems and most recent results in algebraic and arithmetic geometry * Text can serve as an intense introduction for graduate students and those wishing to pursue research in algebraic and arithmetic geometry

number theory and algebra: *Advanced Number Theory* Harvey Cohn, 1980-08-01 A very stimulating book ... in a class by itself. — American Mathematical Monthly Advanced students, mathematicians and number theorists will welcome this stimulating treatment of advanced number theory, which approaches the complex topic of algebraic number theory from a historical standpoint, taking pains to show the reader how concepts, definitions and theories have evolved during the last two centuries. Moreover, the book abounds with numerical examples and more concrete, specific theorems than are found in most contemporary treatments of the subject. The book is divided into three parts. Part I is concerned with background material — a synopsis of elementary number theory (including quadratic congruences and the Jacobi symbol), characters of residue class groups via the structure theorem for finite abelian groups, first notions of integral domains, modules and lattices, and such basis theorems as Kronecker's Basis Theorem for Abelian Groups. Part II discusses ideal theory in quadratic fields, with chapters on unique factorization and units, unique factorization into ideals, norms and ideal classes (in particular, Minkowski's theorem), and class structure in quadratic fields. Applications of this material are made in Part III to class number formulas and primes in arithmetic progression, quadratic reciprocity in the rational domain and the relationship between quadratic forms and ideals, including the theory of composition, orders and genera. In a final concluding survey of more recent developments, Dr. Cohn takes up Cyclotomic Fields and Gaussian Sums, Class Fields and Global and Local Viewpoints. In addition to numerous helpful diagrams and tables throughout the text, appendices, and an annotated bibliography, Advanced Number Theory also includes over 200 problems specially designed to stimulate the spirit of experimentation which has traditionally ruled number theory.

number theory and algebra: *An Illustrated Theory of Numbers* Martin H. Weissman, 2020-09-15 News about this title: — Author Marty Weissman has been awarded a Guggenheim Fellowship for 2020. (Learn more here.) — Selected as a 2018 CHOICE Outstanding Academic Title — 2018 PROSE Awards Honorable Mention An Illustrated Theory of Numbers gives a comprehensive introduction to number theory, with complete proofs, worked examples, and exercises. Its exposition reflects the most recent scholarship in mathematics and its history. Almost 500 sharp illustrations accompany elegant proofs, from prime decomposition through quadratic reciprocity. Geometric and dynamical arguments provide new insights, and allow for a rigorous approach with less algebraic manipulation. The final chapters contain an extended treatment of binary quadratic forms, using Conway's topograph to solve quadratic Diophantine equations (e.g., Pell's equation) and to study reduction and the finiteness of class numbers. Data visualizations introduce the reader to open questions and cutting-edge results in analytic number theory such as the Riemann hypothesis, boundedness of prime gaps, and the class number 1 problem. Accompanying each chapter, historical notes curate primary sources and secondary scholarship to trace the development of number theory within and outside the Western tradition. Requiring only high school algebra and geometry, this text is recommended for a first course in elementary number theory. It is also suitable for mathematicians seeking a fresh perspective on an ancient subject.

number theory and algebra: Algebraic Number Theory Robert L. Long, 1977

number theory and algebra: *An Introduction to Algebraic Number Theory* Takashi Ono, 2012-12-06 This book is a translation of my book Suron Josetsu (An Introduction to Number Theory), Second Edition, published by Shokabo, Tokyo, in 1988. The translation is faithful to the original globally but, taking advantage of my being the translator of my own book, I felt completely free to reform or deform the original locally everywhere. When I sent T. Tamagawa a copy of the First Edition of the original work two years ago, he immediately pointed out that I had skipped the discussion of the class numbers of real quadratic fields in terms of continued fractions and (in a letter dated 2/15/87) sketched his idea of treating continued fractions without writing explicitly continued fractions, an approach he had first presented in his number theory lectures at Yale some years ago. Although I did not follow his approach exactly, I added to this translation a section (Section 4. 9), which nevertheless fills the gap pointed out by Tamagawa. With this addition, the present book covers at least T. Takagi's Shoto Seisuron Kogi (Lectures on Elementary Number

Theory), First Edition (Kyoritsu, 1931), which, in turn, covered at least Dirichlet's Vorlesungen. It is customary to assume basic concepts of algebra (up to, say, Galois theory) in writing a textbook of algebraic number theory. But I feel a little strange if I assume Galois theory and prove Gauss quadratic reciprocity.

number theory and algebra: *Basic Number Theory.* Andre Weil, 2013-12-14 Itpzf}JlOV, li~oxov uoq>ZUJlCJ. 7:WV Al(JX., llpoj1. AE(Jj1. The first part of this volume is based on a course taught at Princeton University in 1961-62; at that time, an excellent set ofnotes was prepared by David Cantor, and it was originally my intention to make these notes available to the mathematical public with only quite minor changes. Then, among some old papers of mine, I accidentally came across a long-forgotten manuscript by ChevaIley, of pre-war vintage (forgotten, that is to say, both by me and by its author) which, to my taste at least, seemed to have aged very welt It contained abrief but essentially com plete account of the main features of c1assfield theory, both local and global; and it soon became obvious that the usefulness of the intended volume would be greatly enhanced if I inc1uded such a treatment of this topic. It had to be expanded, in accordance with my own plans, but its outline could be preserved without much change. In fact, I have adhered to it rather c10sely at some critical points.

number theory and algebra: Computational Algebra and Number Theory Wieb Bosma, Alf van der Poorten, 2013-03-09 Computers have stretched the limits of what is possible in mathematics. More: they have given rise to new fields of mathematical study; the analysis of new and traditional algorithms, the creation of new paradigms for implementing computational methods, the viewing of old techniques from a concrete algorithmic vantage point, to name but a few. Computational Algebra and Number Theory lies at the lively intersection of computer science and mathematics. It highlights the surprising width and depth of the field through examples drawn from current activity, ranging from category theory, graph theory and combinatorics, to more classical computational areas, such as group theory and number theory. Many of the papers in the book provide a survey of their topic, as well as a description of present research. Throughout the variety of mathematical and computational fields represented, the emphasis is placed on the common principles and the methods employed. Audience: Students, experts, and those performing current research in any of the topics mentioned above.

number theory and algebra: Computer Algebra and Polynomials Jaime Gutierrez, Josef Schicho, Martin Weimann, 2015-01-20 Algebra and number theory have always been counted among the most beautiful mathematical areas with deep proofs and elegant results. However, for a long time they were not considered that important in view of the lack of real-life applications. This has dramatically changed: nowadays we find applications of algebra and number theory frequently in our daily life. This book focuses on the theory and algorithms for polynomials over various coefficient domains such as a finite field or ring. The operations on polynomials in the focus are factorization, composition and decomposition, basis computation for modules, etc. Algorithms for such operations on polynomials have always been a central interest in computer algebra, as it combines formal (the variables) and algebraic or numeric (the coefficients) aspects. The papers presented were selected from the Workshop on Computer Algebra and Polynomials, which was held in Linz at the Johann Radon Institute for Computational and Applied Mathematics (RICAM) during November 25-29, 2013, at the occasion of the Special Semester on Applications of Algebra and Number Theory.

Additional Resources

number theory and algebra

Number Theory And Algebra

Number Theory And Algebra

Related Articles

- ohms law phet lab answer key
- petsmart wellness dog food
- persuasive language introduction to ethos pathos and logos answer key

Back to Home