rockafellar convex analysis

Rockafellar Convex Analysis: A Deep Dive into the Fundamentals

Introduction:

Are you fascinated by the elegance and power of optimization problems? Do you yearn to understand the theoretical underpinnings of algorithms that drive modern machine learning and data science? Then you've come to the right place. This comprehensive guide delves into the world of Rockafellar's Convex Analysis, a seminal work that laid the foundation for much of modern convex optimization theory. We'll unpack its core concepts, explore key theorems, and demonstrate its practical applications. This post isn't just a summary; it's a journey into the heart of a mathematical masterpiece, equipping you with a solid understanding of this crucial field. Prepare to unlock the secrets of convexity!

What is Convex Analysis?

Before diving into Rockafellar's contribution, let's establish a foundational understanding. Convex analysis is a branch of mathematics that studies convex sets and convex functions. A set is convex if a line segment connecting any two points within the set remains entirely within the set. Similarly, a function is convex if the line segment connecting any two points on its graph lies above the graph itself. These seemingly simple definitions have profound consequences, leading to powerful properties and algorithms.

Convexity is crucial because it guarantees the existence and uniqueness of solutions in many optimization problems. Unlike non-convex problems, which can have multiple local optima, convex problems possess a single global optimum, making them significantly easier to solve. This characteristic has made convex analysis indispensable in various fields, including:

Machine Learning: Many machine learning algorithms rely on convex optimization for training models, ensuring efficient and accurate results.
Operations Research: Solving problems related to resource allocation, scheduling, and logistics often involves convex optimization techniques.
Economics: Convex analysis plays a crucial role in modeling economic systems and optimizing resource allocation.
Engineering: Design optimization problems in various engineering disciplines frequently leverage the properties of convexity.

Rockafellar's Contribution: A Landmark Text

R. Tyrrell Rockafellar's "Convex Analysis," published in 1970, is considered the definitive text on the subject. It's not an easy read—it demands a strong mathematical background—but it's a rewarding one for those willing to invest the effort. Rockafellar systematically developed the theory of convex analysis, introducing and rigorously proving fundamental theorems and concepts that remain central to the field today. His work unified

and extended previous results, creating a cohesive and powerful framework for understanding and solving convex optimization problems.

Key Concepts in Rockafellar's Convex Analysis:

Convex Sets and Their Properties: Rockafellar meticulously explores the properties of convex sets, including their closures, interiors, and relative interiors. He establishes fundamental results concerning the separation of convex sets and the representation of convex sets as intersections of half-spaces.
Convex Functions and Their Properties: The book dives deep into the properties of convex functions, including subgradients, conjugate functions, and their relationships. The concept of the subgradient, a generalization of the gradient for non-differentiable functions, is central to many optimization algorithms.
Conjugate Functions and Duality: The concept of conjugate functions plays a pivotal role in Rockafellar's framework. Conjugate duality is a powerful tool for analyzing and solving optimization problems, providing alternative perspectives and enabling the development of efficient algorithms.
Subdifferentials and Optimization: The subdifferential, the set of all subgradients at a point, is extensively explored. Rockafellar demonstrates its importance in characterizing optimality conditions and developing iterative algorithms for solving convex optimization problems.
Applications and Extensions: While rigorous in its mathematical treatment, Rockafellar's work doesn't shy away from showcasing the practical applications of convex analysis. He demonstrates how the developed theory can be applied to various optimization problems, providing a bridge between theory and practice.

A Structured Outline of Rockafellar's Convex Analysis:

Book Title: Convex Analysis

Outline:

Introduction: Overview of convexity, its importance, and the book's scope.
Chapter 1: Convex Sets: Definitions, properties, separation theorems, and topological aspects of convex sets.
Chapter 2: Convex Functions: Definitions, properties, subgradients, directional derivatives, and continuity.
Chapter 3: Conjugate Functions: Definition, properties, Fenchel duality, and applications to optimization.
Chapter 4: Subdifferentials: Detailed analysis of subdifferentials, their properties, and relationships to optimality conditions.
Chapter 5: Applications to Optimization: Illustrative examples and applications of convex analysis to various optimization problems.
Chapter 6: Extensions and Advanced Topics: More advanced concepts and extensions of the theory (e.g., generalized convexity).
Conclusion: Summary of key concepts and their significance in the field of optimization.
Appendix: Mathematical preliminaries and notations.

Detailed Explanation of the Outline:

(1) Introduction: This section lays the groundwork, providing a clear definition of convexity and highlighting its importance in optimization. It sets the stage for the more technical chapters to follow.

(2) Chapter 1: Convex Sets: This chapter delves into the fundamental building blocks—convex sets. It rigorously defines convexity and explores various properties, including convex hulls, separation theorems (crucial for characterizing solutions), and topological considerations related to closure and interior points. Understanding this chapter is crucial for grasping subsequent material.

(3) Chapter 2: Convex Functions: Here, the focus shifts to convex functions. This chapter defines convex functions, explores their properties (e.g., continuity, differentiability), introduces the crucial concept of subgradients (generalizing gradients for non-differentiable functions), and examines directional derivatives. Mastering this chapter is essential for understanding optimization algorithms based on subgradients.

(4) Chapter 3: Conjugate Functions: This chapter introduces the powerful concept of conjugate functions, a pivotal tool in duality theory. It explains how to compute conjugate functions, demonstrates their properties, and showcases their role in Fenchel duality, a cornerstone of convex optimization. Understanding conjugate functions is essential for developing advanced optimization algorithms.

(5) Chapter 4: Subdifferentials: This chapter provides an in-depth analysis of subdifferentials—the sets of all subgradients at a given point. Rockafellar meticulously explores their properties, establishes their connections to optimality conditions (necessary and sufficient conditions for a point to be a minimizer), and lays the foundation for many optimization algorithms.

(6) Chapter 5: Applications to Optimization: This chapter bridges the gap between theory and practice. It provides concrete examples of how the theoretical tools developed in the preceding chapters can be applied to solve various optimization problems. It serves as a practical demonstration of the power of convex analysis.

(7) Chapter 6: Extensions and Advanced Topics: This chapter delves into more specialized and advanced topics, exploring extensions of the core concepts and addressing more complex scenarios. This could include generalized notions of convexity or applications to specific types of optimization problems.

(8) Conclusion: This section summarizes the key contributions and highlights the lasting impact of Rockafellar's work on the field of convex optimization.

(9) Appendix: This section provides a comprehensive list of mathematical preliminaries and notations used throughout the book, ensuring clarity and accessibility for the reader.

FAQs:

1. What is the prerequisite knowledge needed to understand Rockafellar's Convex Analysis? A strong background in linear algebra, real analysis, and some familiarity with topology is essential.

2. Is Rockafellar's book suitable for beginners in convex optimization? No, it's a highly advanced text best suited for those with a solid mathematical foundation.

3. What are the main applications of Rockafellar's Convex Analysis? Machine learning, operations research, economics, and engineering.

4. What is the significance of conjugate functions in Rockafellar's work? They are crucial for duality theory and developing efficient optimization algorithms.

5. What are subgradients and why are they important? They are generalizations of gradients for non-differentiable functions and are essential for characterizing optimality conditions.

6. How does Rockafellar's book contribute to the field of optimization? It provides a rigorous and unified framework for understanding and solving convex optimization problems.

7. Are there any online resources that can help in understanding Rockafellar's Convex Analysis? While the book itself is the primary resource, online lecture notes and supplementary materials from universities can be helpful.

8. What are some alternative books on convex analysis that are more accessible to beginners? Several introductory texts offer a gentler approach to the subject.

9. Is it necessary to read the entire book to understand the core concepts? While reading the entire book is ideal, focusing on key chapters and concepts can provide a solid understanding of the fundamentals.

Related Articles:

1. Introduction to Convex Optimization: A beginner-friendly overview of the field.

2. Subgradient Methods in Machine Learning: Explores algorithms using subgradients.

3. Fenchel Duality and its Applications: A deeper dive into the power of conjugate duality.

4. Convex Optimization in Machine Learning Algorithms: Focuses on practical applications in ML.

5. The Role of Convexity in Operations Research: Discusses applications in logistics and resource allocation.

6. Geometric Interpretation of Convex Sets: A visual approach to understanding convex sets.

7. Applications of Convex Analysis in Economics: Explores its role in economic modeling.

8. Solving Convex Optimization Problems with Python: A practical tutorial on implementing algorithms.

9. Advanced Topics in Convex Analysis: Delves into more complex concepts and extensions.

This comprehensive guide provides a solid foundation for understanding Rockafellar's influential work. While the book itself is a challenging read, this article serves as a valuable starting point for those seeking to delve into the fascinating world of convex analysis and its powerful applications. Remember to consult the original text for a complete and rigorous understanding.

rockafellar convex analysis: Convex Analysis R. Tyrrell Rockafellar, 1997-01-12 Topics treat systems of inequalities; Lagrange multipliers; minimax theorems and duality; structures of convex sets and functions; and more. Available for the first time in paperback, Rockafellar's classic study has firmly established a vital area not only for pure mathematics but also for applications to economics and engineering. Readers will find sound knowledge of linear algebra and introductory real analysis a major benefit to the assimilation of this work.

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rockafellar convex analysis: Convex Analysis R. Tyrrell Rockafellar, 1972

rockafellar convex analysis: Convex Analysis and Optimization Dimitri Bertsekas, Angelia Nedic, Asuman Ozdaglar, 2003-03-01 A uniquely pedagogical, insightful, and rigorous treatment of the analytical/geometrical foundations of optimization. The book provides a comprehensive development of convexity theory, and its rich applications in optimization, including duality, minimax/saddle point theory, Lagrange multipliers, and Lagrangian relaxation/nondifferentiable optimization. It is an excellent supplement to several of our books: Convex Optimization Theory (Athena Scientific, 2009), Convex Optimization Algorithms (Athena Scientific, 2015), Nonlinear Programming (Athena Scientific, 2016), Network Optimization (Athena Scientific, 1998), and Introduction to Linear Optimization (Athena Scientific, 1997). Aside from a thorough account of convex analysis and optimization, the book aims to restructure the theory of the subject, by introducing several novel unifying lines of analysis, including: 1) A unified development of minimax theory and constrained optimization duality as special cases of duality between two simple geometrical problems. 2) A unified development of conditions for existence of solutions of convex optimization problems, conditions for the minimax equality to hold, and conditions for the absence of a duality gap in constrained optimization. 3) A unification of the major constraint qualifications allowing the use of Lagrange multipliers for nonconvex constrained optimization, using the notion of constraint pseudonormality and an enhanced form of the Fritz John necessary optimality conditions. Among its features the book: a) Develops rigorously and comprehensively the theory of convex sets and functions, in the classical tradition of Fenchel and Rockafellar b) Provides a geometric, highly visual treatment of convex and nonconvex optimization problems, including existence of solutions, optimality conditions, Lagrange multipliers, and duality c) Includes an insightful and comprehensive presentation of minimax theory and zero sum games, and its connection with duality d) Describes dual optimization, the associated computational methods, including the novel incremental subgradient methods, and applications in linear, quadratic, and integer programming e) Contains many examples, illustrations, and exercises with complete solutions (about 200 pages) posted at the publisher's web site http://www.athenasc.com/convexity.html

rockafellar convex analysis: Convex Analysis and Monotone Operator Theory in Hilbert Spaces Heinz H. Bauschke, Patrick L. Combettes, 2017-02-28 This reference text, now in its second edition, offers a modern unifying presentation of three basic areas of nonlinear analysis: convex analysis, monotone operator theory, and the fixed point theory of nonexpansive operators. Taking a unique comprehensive approach, the theory is developed from the ground up, with the rich connections and interactions between the areas as the central focus, and it is illustrated by a large number of examples. The Hilbert space setting of the material offers a wide range of applications while avoiding the technical difficulties of general Banach spaces. The authors have also drawn upon recent advances and modern tools to simplify the proofs of key results making the book more accessible to a broader range of scholars and users. Combining a strong emphasis on applications with exceptionally lucid writing and an abundance of exercises, this text is of great value to a large audience including pure and applied mathematicians as well as researchers in engineering, data science, machine learning, physics, decision sciences, economics, and inverse problems. The second edition of Convex Analysis and Monotone Operator Theory in Hilbert Spaces greatly expands on the first edition, containing over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators including two sections on proximity operators of matrix functions, in addition to several new sections distributed throughout the original chapters. Many existing results have been improved, and the list of references has been updated. Heinz H. Bauschke is a Full Professor of Mathematics at the Kelowna campus of the University of British Columbia, Canada. Patrick L. Combettes, IEEE Fellow, was on the faculty of the City University of New York and of Université Pierre et Marie Curie – Paris 6 before joining North Carolina State University as a Distinguished Professor of Mathematics in 2016.

rockafellar convex analysis: *Convex Optimization* Stephen P. Boyd, Lieven Vandenberghe, 2004-03-08 Convex optimization problems arise frequently in many different fields. This book provides a comprehensive introduction to the subject, and shows in detail how such problems can be solved numerically with great efficiency. The book begins with the basic elements of convex sets and functions, and then describes various classes of convex optimization problems. Duality and approximation techniques are then covered, as are statistical estimation techniques. Various geometrical problems are then presented, and there is detailed discussion of unconstrained and constrained minimization problems, and interior-point methods. The focus of the book is on recognizing convex optimization problems and then finding the most appropriate technique for solving them. It contains many worked examples and homework exercises and will appeal to students, researchers and practitioners in fields such as engineering, computer science, mathematics, statistics, finance and economics.

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(Athena Scientific, 1998).

rockafellar convex analysis: Convex Analysis and Variational Problems Ivar Ekeland, Roger Temam, 1999-12-01 This book contains different developments of infinite dimensional convex programming in the context of convex analysis, including duality, minmax and Lagrangians, and convexification of nonconvex optimization problems in the calculus of variations (infinite dimension). It also includes the theory of convex duality applied to partial differential equations; no other reference presents this in a systematic way. The minmax theorems contained in this book have many useful applications, in particular the robust control of partial differential equations in finite time horizon. First published in English in 1976, this SIAM Classics in Applied Mathematics edition contains the original text along with a new preface and some additional references.

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rockafellar convex analysis: An Easy Path to Convex Analysis and Applications Boris S. Mordukhovich, Nguyen Mau Nam, 2013-12-01 Convex optimization has an increasing impact on many areas of mathematics, applied sciences, and practical applications. It is now being taught at many universities and being used by researchers of different fields. As convex analysis is the mathematical f

rockafellar convex analysis: *Convex Analysis and Nonlinear Optimization* Jonathan Borwein, Adrian S. Lewis, 2010-05-05 Optimization is a rich and thriving mathematical discipline, and the underlying theory of current computational optimization techniques grows ever more sophisticated. This book aims to provide a concise, accessible account of convex analysis and its applications and extensions, for a broad audience. Each section concludes with an often extensive set of optional exercises. This new edition adds material on semismooth optimization, as well as several new proofs.

rockafellar convex analysis: Conjugate Duality and Optimization R. Tyrrell Rockafellar, 1974-01-01 Provides a relatively brief introduction to conjugate duality in both finite- and infinite-dimensional problems. An emphasis is placed on the fundamental importance of the concepts of Lagrangian function, saddle-point, and saddle-value. General examples are drawn from nonlinear programming, approximation, stochastic programming, the calculus of variations, and optimal control.

rockafellar convex analysis: Conjugate Duality in Convex Optimization Radu Ioan-Bot, 2010-02-04 The results presented in this book originate from the last decade research work of the author in the ?eld of duality theory in convex optimization. The reputation of duality in the optimization theory comes mainly from the major role that it plays in formulating necessary and suf?cient optimality conditions and, consequently, in generatingdifferent algorithmic approachesfor solving mathematical programming problems. The investigations made in this work prove the importance of the duality theory beyond these aspects and emphasize its strong connections with different topics in convex analysis, nonlinear analysis, functional analysis and in the theory of monotone operators. The ?rst part of the book brings to the attention of the reader the perturbation approach as a fundamental tool for developing the so-called conjugate duality t- ory. The classical Lagrange and Fenchel duality approaches are particular instances of this general concept. More than that, the generalized interior point regularity conditions stated in the past for the two mentioned situations turn out to be p- ticularizations of the ones given in this general setting. In our investigations, the perturbationapproachrepresentsthestartingpointforderivingnewdualityconcepts for several classes of convex optimization problems. Moreover, via this approach, generalized Moreau–Rockafellar formulae are provided and, in connection with them, a new class of regularity conditions, called closedness-type conditions, for both stable strong duality and strong duality is introduced. By stable strong duality we understand the situation in which strong duality still holds

whenever perturbing the objective function of the primal problem with a linear continuous functional.

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rockafellar convex analysis: *Statistical Inference Via Convex Optimization* Anatoli Juditsky, Arkadi Nemirovski, 2020-04-07 This authoritative book draws on the latest research to explore the interplay of high-dimensional statistics with optimization. Through an accessible analysis of fundamental problems of hypothesis testing and signal recovery, Anatoli Juditsky and Arkadi Nemirovski show how convex optimization theory can be used to devise and analyze near-optimal statistical inferences. Statistical Inference via Convex Optimization is an essential resource for optimization specialists who are new to statistics and its applications, and for data scientists who want to improve their optimization methods. Juditsky and Nemirovski provide the first systematic treatment of the statistical techniques that have arisen from advances in the theory of optimization. They focus on four well-known statistical problems—sparse recovery, hypothesis testing, and recovery from indirect observations of both signals and functions of signals—demonstrating how they can be solved more efficiently as convex optimization problems. The emphasis throughout is on achieving the best possible statistical performance. The construction of inference routines and the quantification of their statistical performance are given by efficient computation rather than by analytical derivation typical of more conventional statistical approaches. In addition to being computation-friendly, the methods described in this book enable practitioners to handle numerous situations too difficult for closed analytical form analysis, such as composite hypothesis testing and signal recovery in inverse problems. Statistical Inference via Convex Optimization features exercises with solutions along with extensive appendixes, making it ideal for use as a graduate text.

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or too closely attached to numerical algorithms. Some exercises were included and finally the index has been considerably enriched, making it an excellent choice for the purpose of learning and teaching.

rockafellar convex analysis: Convex Analysis and Beyond Boris S. Mordukhovich, Nguyen Mau Nam, 2022-04-24 This book presents a unified theory of convex functions, sets, and set-valued mappings in topological vector spaces with its specifications to locally convex, Banach and finite-dimensional settings. These developments and expositions are based on the powerful geometric approach of variational analysis, which resides on set extremality with its characterizations and specifications in the presence of convexity. Using this approach, the text consolidates the device of fundamental facts of generalized differential calculus to obtain novel results for convex sets, functions, and set-valued mappings in finite and infinite dimensions. It also explores topics beyond convexity using the fundamental machinery of convex analysis to develop nonconvex generalized differentiation and its applications. The text utilizes an adaptable framework designed with researchers as well as multiple levels of students in mind. It includes many exercises and figures suited to graduate classes in mathematical sciences that are also accessible to advanced students in economics, engineering, and other applications. In addition, it includes chapters on convex analysis and optimization in finite-dimensional spaces that will be useful to upper undergraduate students, whereas the work as a whole provides an ample resource to mathematicians and applied scientists, particularly experts in convex and variational analysis, optimization, and their applications.

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problems. A function is quasiconvex if its lower level sets are convex. Here again, the geo metrical structure of the level sets implies some continuity and differentiability properties for quasiconvex functions. Optimality conditions and duality can be derived for optimization problems involving such functions as well. Over a period of about fifty years, quasiconvex and other generalized convex functions have been considered in a variety of fields including economies, man agement science, engineering, probability and applied sciences in accordance with the need of particular applications. During the last twenty-five years, an increase of research activities in this field has been witnessed. More recently generalized monotonicity of maps has been studied. It relates to generalized convexity off unctions as monotonicity relates to convexity. Generalized monotonicity plays a role in variational inequality problems, complementarity problems and more generally, in equilibrium prob lems.

rockafellar convex analysis: *Nonsmooth Optimization and Related Topics* F.H. Clarke, Vladimir F. Dem'yanov, F. Giannessi, 2013-11-11 This volume contains the edited texts of the lect. nres presented at the International School of Mathematics devoted to Nonsmonth Optimization, held from . June 20 to July I, 1988. The site for the meeting was the Ettore ~Iajorana Centre for Sci entific Culture in Erice, Sicily. In the tradition of these meetings the main purpose was to give the state-of-the-art of an important and growing field of mathematics, and to stimulate interactions between finite-dimensional and infinite-dimensional op timization. The School was attended by approximately 80 people from 23 countries; in particular it was possible to have some distinguished lecturers from the SO\·iet Union, whose research institutions are here gratt-fnlly acknowledged. Besides the lectures, several seminars were delivered; a special s·~ssion was devoted to numerical computing aspects. The result was a broad exposure. gi ·. ring a deep knowledge of the present research tendencies in the field. We wish to express our appreciation to all the participants. Special mention 5hould be made of the Ettorc ;. . Iajorana Centre in Erice, which helped provide a stimulating and rewarding experience, and of its staff which was fundamental for the success of the meeting. j\, loreover, WP want to extend uur deep appreci

rockafellar convex analysis: Advances in Convex Analysis and Global Optimization Nicolas Hadjisavvas, Panos M. Pardalos, 2013-12-01 There has been much recent progress in global optimization algo rithms for nonconvex continuous and discrete problems from both a theoretical and a practical perspective. Convex analysis plays a fun damental role in the analysis and development of global optimization algorithms. This is due essentially to the fact that virtually all noncon vex optimization problems can be described using differences of convex functions and differences of convex sets. A conference on Convex Analysis and Global Optimization was held during June 5 -9, 2000 at Pythagorion, Samos, Greece. The conference was honoring the memory of C. Caratheodory (1873-1950) and was en dorsed by the Mathematical Programming Society (MPS) and by the Society for Industrial and Applied Mathematics (SIAM) Activity Group in Optimization. The conference was sponsored by the European Union (through the EPEAEK program), the Department of Mathematics of the Aegean University and the Center for Applied Optimization of the University of Florida, by the General Secretariat of Research and Tech nology of Greece, by the Ministry of Education of Greece, and several local Greek government agencies and companies. This volume contains a selective collection of refereed papers based on invited and contribut ing talks presented at this conference. The two themes of convexity and global optimization pervade this book. The conference provided a forum for researchers working on different aspects of convexity and global opti mization to present their recent discoveries, and to interact with people working on complementary aspects of mathematical programming.

rockafellar convex analysis: Convex Optimization Sébastien Bubeck, 2015-11-12 This monograph presents the main complexity theorems in convex optimization and their corresponding algorithms. It begins with the fundamental theory of black-box optimization and proceeds to guide the reader through recent advances in structural optimization and stochastic optimization. The presentation of black-box optimization, strongly influenced by the seminal book by Nesterov, includes the analysis of cutting plane methods, as well as (accelerated) gradient descent schemes. Special attention is also given to non-Euclidean settings (relevant algorithms include Frank-Wolfe,

mirror descent, and dual averaging), and discussing their relevance in machine learning. The text provides a gentle introduction to structural optimization with FISTA (to optimize a sum of a smooth and a simple non-smooth term), saddle-point mirror prox (Nemirovski's alternative to Nesterov's smoothing), and a concise description of interior point methods. In stochastic optimization it discusses stochastic gradient descent, mini-batches, random coordinate descent, and sublinear algorithms. It also briefly touches upon convex relaxation of combinatorial problems and the use of randomness to round solutions, as well as random walks based methods.

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rockafellar convex analysis: From Analysis to Visualization David H. Bailey, Naomi Simone Borwein, Richard P. Brent, Regina S. Burachik, Judy-anne Heather Osborn, Brailey Sims, Qiji J. Zhu, 2020-03-16 Students and researchers from all fields of mathematics are invited to read and treasure this special Proceedings. A conference was held 25 –29 September 2017 at Noah’s On the Beach, Newcastle, Australia, to commemorate the life and work of Jonathan M. Borwein, a mathematician extraordinaire whose untimely passing in August 2016 was a sorry loss to mathematics and to so many members of its community, a loss that continues to be keenly felt. A polymath, Jonathan Borwein ranks among the most wide ranging and influential mathematicians of the last 50 years, making significant contributions to an exceptional diversity of areas and substantially expanding the use of the computer as a tool of the research mathematician. The contributions in this commemorative volume probe Dr. Borwein's ongoing legacy in areas where he did some of his most outstanding work: Applied Analysis, Optimization and Convex Functions; Mathematics Education; Financial Mathematics; plus Number Theory, Special Functions and Pi, all tinged by the double prisms of Experimental Mathematics and Visualization, methodologies he championed.

rockafellar convex analysis: Delay-Adaptive Linear Control Yang Zhu, Miroslav Krstic, 2020-04-28 Basic predictor feedback for single-input systems -- Basic idea of adaptive control for single-input systems -- Single-input systems with full relative degree -- Single-input systems with arbitrary relative degree -- Exact predictor feedback for multi-input systems -- Full-state feedback of uncertain multi-input systems -- Output feedback of uncertain multi-input systems -- Output feedback of systems with uncertain delays, parameters and ODE state -- Predictor feedback for uncertainty-free systems -- Predictor feedback of uncertain single-input systems -- Predictor feedback of uncertain multi-input systems.

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subdifferential, variational characterizations of convexity, and the study of Chebychev sets. At the same time, the underlying geometrical meaning of all the involved concepts and operations is highlighted and duly emphasized. A notable feature of the book is its unifying methodology, as well as the novelty of providing an alternative or complementary view to the traditional one in which the discipline is presented to students and researchers. This textbook can be used for courses on optimization, convex and variational analysis, addressed to graduate and post-graduate students of mathematics, and also students of economics and engineering. It is also oriented to provide specific background for courses on optimal control, data science, operations research, economics (game theory), etc. The book represents a challenging and motivating development for those experts in functional analysis, convex geometry, and any kind of researchers who may be interested in applications of their work.

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Mordukhovich, Nguyen Mau Nam, 2023-06-16 This book examines the most fundamental parts of convex analysis and its applications to optimization and location problems. Accessible techniques of variational analysis are employed to clarify and simplify some basic proofs in convex analysis and to build a theory of generalized differentiation for convex functions and sets in finite dimensions. The book serves as a bridge for the readers who have just started using convex analysis to reach deeper topics in the field. Detailed proofs are presented for most of the results in the book and also included are many figures and exercises for better understanding the material. Applications provided include both the classical topics of convex optimization and important problems of modern convex optimization, convex geometry, and facility location.

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